LESSON 1

Points, Lines, Rays, and Line Segments

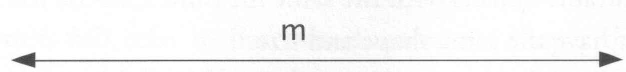
Point - Geometry is the measure of the earth. "Geo" means earth and "metry" means measure. To measure the earth, we need to break it into smaller, more manageable pieces. The smallest unit of measure is an imaginary piece called a *point*. It has no measurable size, only position, or location. We can't measure its width or length, so it has no dimensions, or is zero-dimensional. To show something that is so small you can't really see it, we draw a dot. The dot is the "graph" of the point; it represents the point. We call it "point A" and label it with a capital, or uppercase, letter.

Figure 1 . A

Line - Using the point as the building block, consider a lot of connected points. A *line* is defined as an infinite (∞) number of connected points. It can be curved or straight. For our purposes, when we refer to a line, it will be straight unless mentioned otherwise. Two points that are contained in the same line are said to be *collinear*. Since a line is as wide as a point, it has no width. But a line does have one dimension, which is length, so it is one-dimensional. A line is drawn, or "graphed," with arrows at both ends to show that it goes on indefinitely, or infinitely.

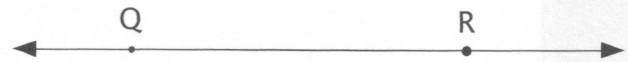
To label a line, use a lowercase letter or choose two points (represented with uppercase letters) in the line. Figure 2 we call "line m" and figure 3 "line QR," or $\overleftrightarrow{QR}$. In figure 3, the order of the points is not important. It could also be named "line RQ," or $\overleftrightarrow{RQ}$.

Figure 2



CoLinear - Exists within the same Line

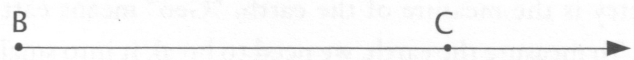
Figure 3



Some other figures which relate to the line are rays and line segments.

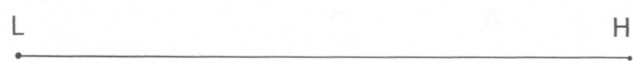
Ray - A *ray* is often referred to as one-half of a line. It has a specific starting point at one end, called the *endpoint*, or *origin*, and then proceeds infinitely in the other direction. Think of a ray as a flashlight or laser beam. Figure 4 is labeled as $\overrightarrow{BC}$ and read as "ray BC." When you label a ray, the order of the points is very important. The first letter is always the origin.

Figure 4



Line Segment - A *line segment* is a finite, or measurable, piece of a line. It is not infinite. It proceeds from one endpoint to another endpoint. It has a specific length. Figure 5 is labeled as $\overline{LH}$ and read as "line segment LH," or "segment LH."

Figure 5



Symbols - When speaking of two shapes being exactly the same, we say they are *congruent*. The symbol for congruent is ≅. It comes from putting ~ and = together. The symbol ~ by itself means similar, or the same shape but not exactly the same size. Two squares have exactly the same shape but may have different measurements. Consider a house and a picture of a house. They have the same shape but are not the exact size, so they are *similar*.

The equals sign (=) means exactly the same length, or equal, and is used if two line segments are the same measurable length. Putting the symbols together means exactly the same shape and size, which gives us congruent (≅). Use the equals sign for measurable objects with the same measure. Choose the congruent sign for objects that have the same shape and size.

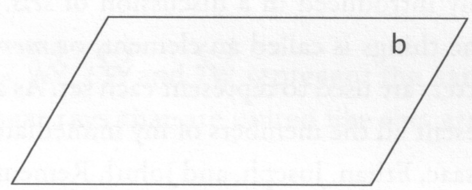
AB means the distance between A and B. If there is no symbol over the AB as in a ray, line, or line segment, then AB is the distance between the two points.

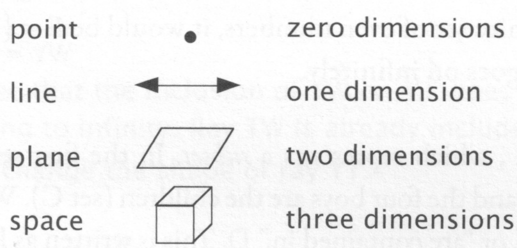
LESSON 2

Planes and Sets

Plane - A *plane* is an infinite number of connected lines lying in the same flat surface. A plane has length and width, so it has two dimensions. A plane can be curved, as in a rolled up piece of paper, but for our purposes it will be flat. We graph a plane by drawing a parallelogram and labeling it with a lowercase letter. In figure 1, the plane is referred to as "plane *b*."

Figure 1





Two lines that intersect are contained in the same plane. Two lines that lie in the same plane are said to be *coplanar*. The intersection of two planes is a line.

Most of our attention will refer to flat, two-dimensional shapes that lie in a plane. This is called *plane geometry*. Three-dimensional geometry, with length, width, and height (or depth), pertaining to space and solids, is called space geometry, or *solid geometry*. Solid geometry applies to volume, as of a cube, cylinder, pyramid, cone, or sphere, all of which will be covered later.

Set Symbolism - In the beginning of our study, we are being introduced to new concepts and shapes. We are also learning new symbolism and vocabulary. In lesson 1, we learned similar (~) and congruent (≅). In this lesson, there are five new symbols. They are ∩, ∪, ∅, ⊂, and { }.

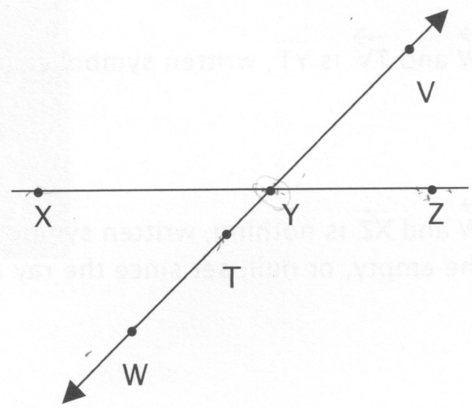
Intersection - ∩	This is where two or more things or groups meet or overlap.
Union - ∪	This is where two or more things or groups are combined.
Empty set, or null set - ∅	This means there is no possible answer.
Subset - ⊂	This means one set is a subset of another.
Set - { }	These are the symbols that represent a set.

These symbols are usually introduced in a discussion of *sets*. A set is a collection of things. Each of the things is called an element, or *member* of the set. Normally in math, capital letters are used to represent each set. As an example, the set D, for Demme, will represent all the members of my immediate family. Set D, or just D = {Steve, Sandra, Isaac, Ethan, Joseph, and John}. Remember that braces ({ })are used as the symbol for a set. This is a finite set consisting of six members. If I were to show the infinite set of even numbers, it would be E = {2, 4, 6. . .} with the dots meaning that it goes on infinitely.

Another symbol is ⊂ , which represents a *subset*. In the first set D, Steve and Sandra are parents (set P) and the four boys are the children (set C). We could say the parents (P) are a subset of, or "are contained in," D. This is written as P ⊂ D. If we are speaking of the children, it would be C ⊂ D, which is the children included in the Demmes. We also might describe the Demmes in terms of five males and one female. If the question is asked, "How many female children?" and there aren't any, then the answer can be written as { } (empty set) or ∅ (empty or null set).

Intersection and Union - Let's use geometry to illustrate *intersection* (∩) and *union* (∪) in the following examples. Refer to figure 2.

Figure 2



Example 1

The union of line segment $\overline{XY}$ and line segment $\overline{YZ}$ is line segment $\overline{XZ}$. Using the symbols, $\overline{XY} \cup \overline{YZ} = \overline{XZ}$. Notice when referring to union, the two elements are combined, or united.

Example 2

$\overrightarrow{TW} \cup \overrightarrow{TY} = \overleftrightarrow{WY}$. ($\overleftrightarrow{TY}$ and $\overleftrightarrow{TW}$ represent the same line as $\overleftrightarrow{WY}$.)
Two collinear rays that are united like this are a line.

Example 3

$\overrightarrow{YT} \cup \overrightarrow{TW} = \overrightarrow{YW}$
Do you see that the inclusion of $\overrightarrow{TW}$ on $\overrightarrow{YT}$ does not change $\overrightarrow{YT}$ since it is already going to infinity. Ray TW is already included in ray YT; joining them does not change the shape of ray YT.

Example 4

The intersection (∩) of $\overline{XY}$ and $\overline{TY}$ is point Y. This intersection is what they share, or have in common. It is symbolized as $\overline{XY} \cap \overline{TY} = Y$.

Example 5

The intersection of $\overrightarrow{YW}$ and $\overrightarrow{TV}$ is $\overline{YT}$, written symbolically as $\overrightarrow{YW} \cap \overrightarrow{TV} = \overline{YT}$.

Example 6

The intersection of $\overrightarrow{TW}$ and $\overline{XZ}$ is nothing, written symbolically as $\overrightarrow{TW} \cap \overline{XZ} = \varnothing$ or { }. The answer is the empty, or null, set since the ray and line segment don't intersect.

Example 7

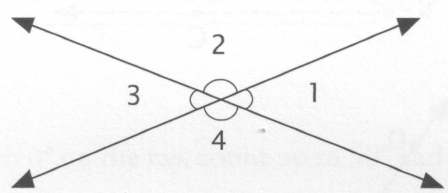
Using subsets: $\overline{XY} \subset \overline{XZ}$ and $\overline{TY} \subset \overrightarrow{TY}$ since the first members are contained in the second members.

LESSON 3

Angles

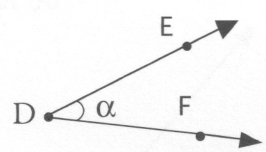
Angles - If two lines intersect, the opening, or space, between the lines is referred to as an *angle*. In figure 1 there are four angles shown by the arcs. The angles are named 1, 2, 3, and 4.

Figure 1



In figure 2 we are focusing on just one angle, which is made by drawing two rays with a common endpoint. This endpoint, or origin, is called the *vertex*. (The plural of vertex is "vertices.")

Figure 2



The rays are $\overrightarrow{DE}$ and $\overrightarrow{DF}$. The angle is labeled with either a number as in figure 1 or a lowercase Greek letter as in figure 2. We'll call this figure $\angle\alpha$ ("angle alpha"). Another way to identify this angle is by picking one point on each ray and the vertex, so: $\angle EDF$ or $\angle FDE$. Notice the point labeling the vertex is always in the middle.

Measuring Angles - At this point, it would be a good idea to practice reading angles, so you can easily identify them. It is also a good time to meet a protractor, which is used to measure angles. You probably already know how to use a ruler. *Rulers* measure length, but *protractors* measure angles.

Refer to figure 3, and measure the length of the line segments AB and BC, which are parts of the two rays forming ∠ABC.

Angles are measured in *degrees*. The wider the gap, or opening, the more degrees. The smaller the opening, the fewer degrees there are. The "measure of an angle" is represented by "m∠." To represent "the measure of angle one is thirty degrees," write "m∠1 = 30°."

Using a protractor, find m∠ABC in figure 3. Then find m∠DEF and m∠GHJ in figures 4 and 5.

Figure 3

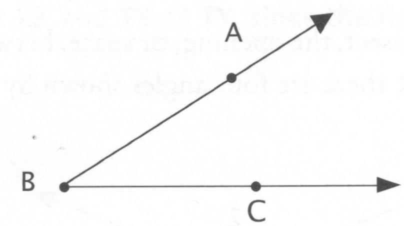


Figure 4

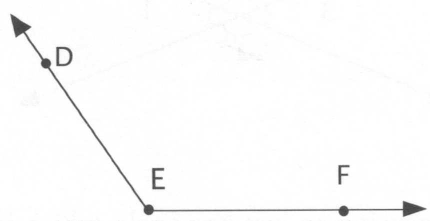
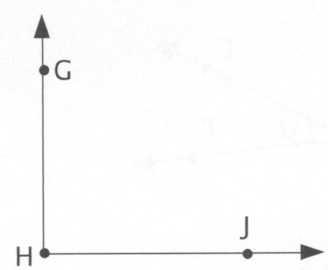


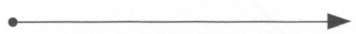
Figure 5



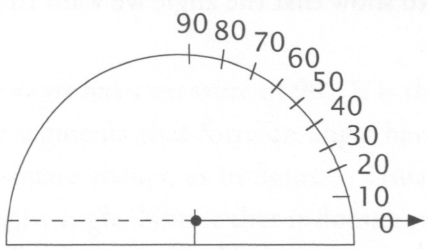
Drawing Angles - Now practice drawing angles with your ruler or straightedge and your protractor.

Draw ∠QRS with measure 50°.

1. Draw a straight line with one endpoint (a ray).

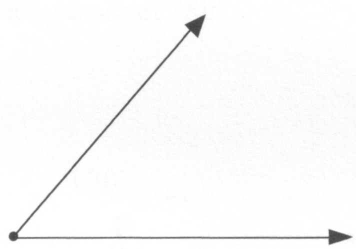


2. Then place your protractor on the line with the cross hairs on the endpoint, and the line going through 0° and 180°.

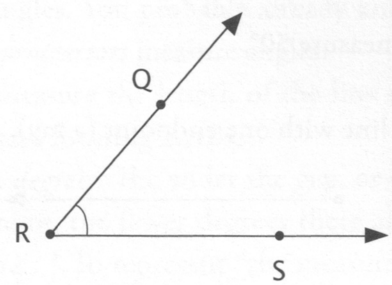


3. Beginning with 0° on the ray, count up to 50° and put a mark on your paper by 50°.

4. Remove the protractor, and using your straightedge, connect the vertex with the mark at 50°.



5. Label the angle with points.



We placed a small arc to show that the angle we want to refer to is inside the rays and not outside them.

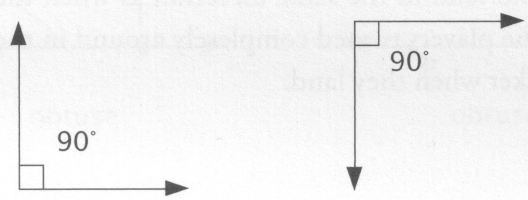
6. Then we write the angle accompanied by the measure: "m∠QRS = 50°," which symbolizes "the measure of angle QRS is 50 degrees."

LESSON 4

Types of Angles

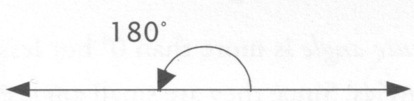
Right Angle - A *right angle* has a measure of 90°. It is the angle used most often. When two rays or line segments that form an angle have a measure of 90°, they form a right angle, or square corner, as in figure 1. Usually a box at the vertex is used to represent the right angle. Notice that it doesn't matter where the angle is, but only that its measure is 90°.

Figure 1



Straight Angle - A *straight angle* is a lesser known angle, which is difficult to think of as an angle. It has a measure of 180°.

Figure 2



You sometimes hear of a car that skidded on ice and did a "one-eighty," meaning the car was going in one direction and then spun around so it was pointing in the opposite direction. This expression comes from the fact that the car spun 180°.

Figure 3 is a top view of this skid, and figure 4 is the side view.

Figure 3

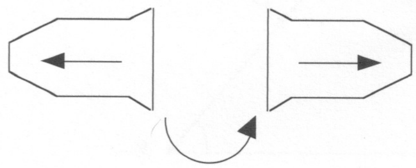
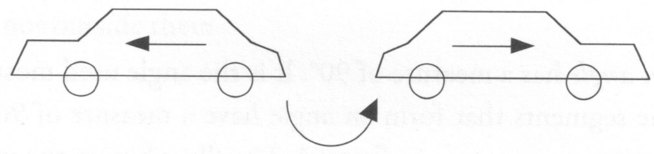
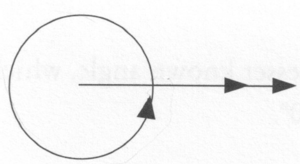


Figure 4



360° Angle - Now in basketball, there are some players that run, jump, turn completely around, and land in the same direction as when they started. This is called a "360" since the players turned completely around in the air, or *360°*, and are still facing the basket when they land.

Figure 5

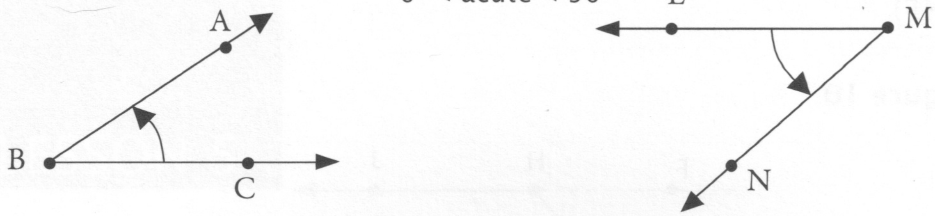


All the way around is 360˚.

Acute Angle - An *acute angle* is more than 0° but less then 90°. Most of the angles you see are acute angles. Since they are small angles, it helps me to remember the name by thinking of "cute." Figure 6 shows two acute angles.

Figure 6

0° < acute < 90°

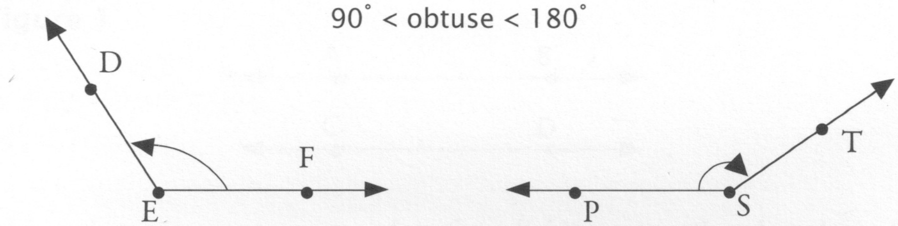


acute   acute

Obtuse Angle - An *obtuse angle* is larger than 90° and less than 180°. See figure 7.

Figure 7

90° < obtuse < 180°

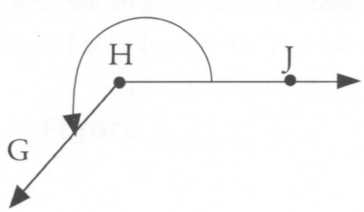


obtuse   obtuse

Reflex Angle - A *reflex angle* is larger than 180° and less than 360°. See figure 8.

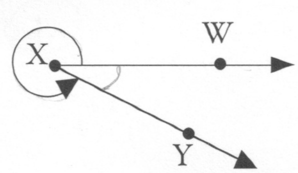
180° < reflex < 360°

Figure 8



reflex

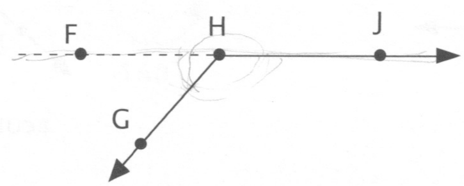
Figure 9



reflex

Measure a Reflex Angle - To measure a reflex angle, you have two options. The first is to measure either the obtuse angle, as in figure 8, or the acute angle as in figure 9, and subtract from 360°. The second option, as shown in figure 10, is to extend one ray to point F. Then measure ∠FHG and add it to 180° (the measure of ∠JHF).

Figure 10



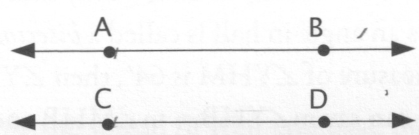
LESSON 5

Parallel and Perpendicular Lines

with Midpoints and Bisectors

Parallel Lines - *Parallel lines* are defined as two lines in the same plane that never intersect. With set symbolism, we could say of figure 1: $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \varnothing$.

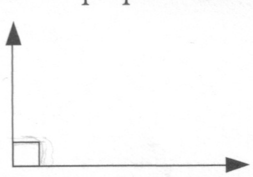
Figure 1



Although not exact, some examples of parallel lines are railroad tracks and the opposite lanes of an interstate highway with a median. If you use lined paper, the lines should be parallel. You hope, when you build a house, that the planes representing the first floor and the ceiling are parallel! The symbol for parallel is ||. We think of this symbol as representative of the two *l*s in the word parallel. In describing figure 1, we say $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Perpendicular Lines - Two lines or rays or line segments that intersect and form a right angle are called *perpendicular lines.* When we have perpendicular lines, we draw a little square where the lines meet to indicate they form a right angle (as in lesson 4). The symbol for perpendicular is $\perp$.

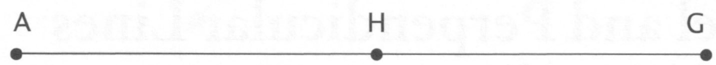
Figure 2



Examples of perpendicular lines may be found in many places. A few obvious ones are the intersection of two roads in a town, where the walls in a house meet, or telephone poles and their cross trees. Perpendicular lines are very important in construction. Contractors call this "making things square." Walls, doors, windows, and corners all have to be square!

Midpoint - Recall a line segment, which has specific length and two endpoints. Of all the points, the one in the very middle is the *midpoint.* It divides the line segment into two line segments that have the same length, or are congruent. In figure 3, note the relationship between $\overline{AH}$ and $\overline{GH}$.

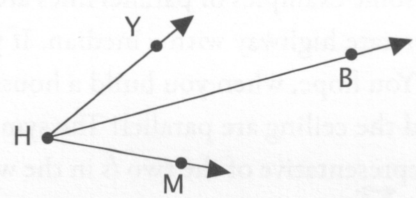
Figure 3



If H is the midpoint, we can conclude that $\overline{AH} \cong \overline{HG}$ and AH = HG. The distance between A and H is the same as the distance between H and G.

Bisector - If an angle is cut exactly in half, we say that it has been *bisected,* and the line or ray which cuts an angle in half is called a *bisector*. In figure 4, ∠YHM is bisected by HB. If the measure of ∠YHM is 64°, then ∠YHB and ∠MHB would each measure 32°. So we can say m∠YHB = m∠MHB and ∠YHB ≅ ∠MHB

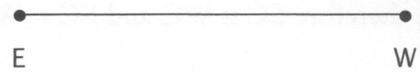
Figure 4



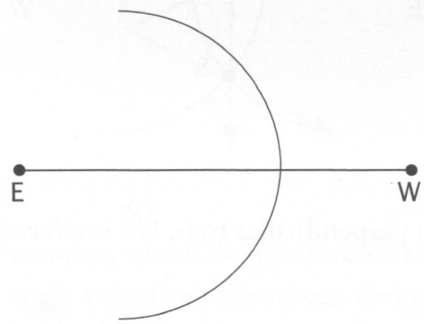
Constructing a Perpendicular Bisector - Now let's use a compass to find the midpoint and bisector of a line segment.

Example 1

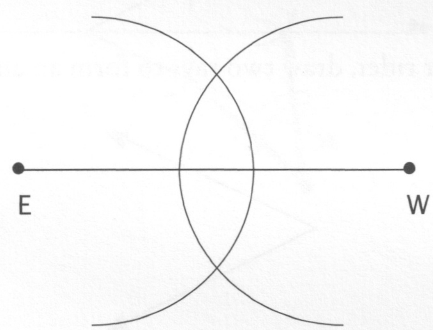
1. Draw a line segment two inches long and label the endpoints E and W.



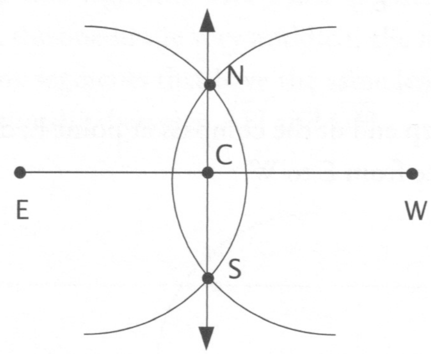
2. Placing the sharp end of the compass at point E, draw an arc that is over half the distance from E to W.



3. Without changing the spread of your compass, move the sharp end to W and draw another arc.



4. Notice the points where the arcs meet above and below $\overline{EW}$. We'll label the points N and S. When we draw a line through N and S, it intersects $\overline{EW}$ at point C. Point C is the midpoint of $\overline{EW}$; therefore $\overline{EC} \cong \overline{WC}$ and EC = WC. Notice also that $\overleftrightarrow{NS} \perp \overleftrightarrow{EW}$.

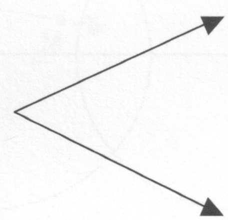


Since $\overleftrightarrow{NS}$ bisects $\overline{EW}$ and is also perpendicular to it, $\overleftrightarrow{NS}$ is referred to as a *perpendicular bisector.*

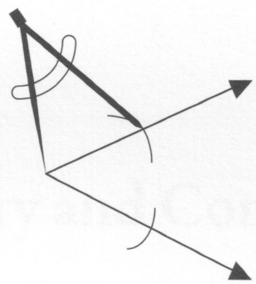
Bisecting an Angle - Now we'll construct the bisector of an angle using only a straight edge and a compass.

Example 2

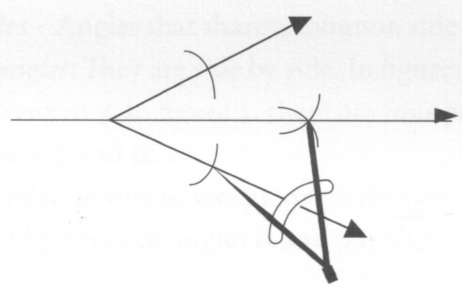
1. Using a straight edge or ruler, draw two rays to form an angle.



2. Set your compass so the distance between the point and the pencil is greater than half the length of either ray. With the compass point on the vertex of the angle, draw an arc that will intersect both rays.



3. Putting the point of the compass where the arcs intersect the rays, make two more arcs that intersect inside the angle away from the vertex. Connect that point of intersection with the vertex of the angle by drawing a line or ray. This is the line or ray that bisects the angle, and it is called the bisector.

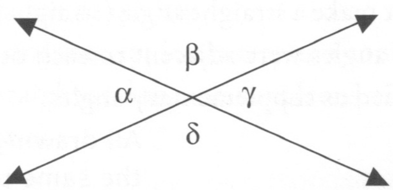


LESSON 6

Supplementary and Complementary Angles

Greek Letters

Figure 1



α = alpha

β = beta

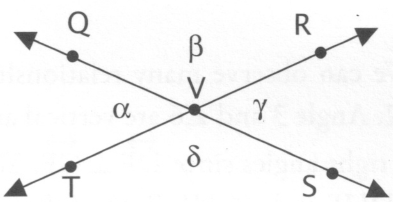
γ= gamma

δ= delta

Adjacent Angles - Angles that share a common side and have the same origin are called *adjacent angles*. They are side by side. In figure 1, α is adjacent to both β and δ. It is not adjacent to γ. In figure 1, there are four pairs of adjacent angles: α and β, β and γ, γ and δ, δ and α.

In figure 2, we added points so we can name the rays that form the angles. The common side shared by adjacent angles α and β is $\overrightarrow{VQ}$.

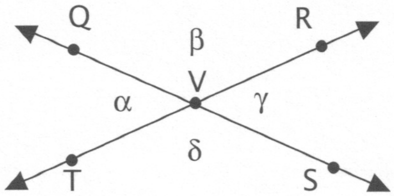
Figure 2



Given: $\overleftrightarrow{RT} \cap \overleftrightarrow{QS} = V$

Vertical Angles - Notice that $\angle\gamma$ is opposite $\angle\alpha$. Angles that share a common origin and are opposite each other are called vertical angles. They have the same measure and are congruent. $\angle\beta$ and $\angle\delta$ are also vertical angles.

Figure 2 (from previous page)



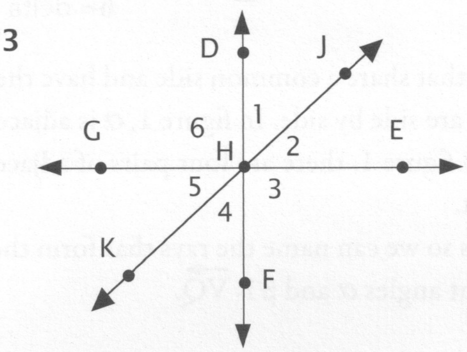
Given: $\overleftrightarrow{RT} \cap \overleftrightarrow{QS} = V$

If m∠β is 115°, then m ∠δ is also 115°. If this is true, then do we have enough information to find m ∠α? We know from the information given in figure 2 that $\overleftrightarrow{RT}$ and $\overleftrightarrow{QS}$ are lines. Therefore, ∠RVT is a straight angle and has a measure of 180°. If ∠RVQ (∠β) is 115°, then ∠QVT (∠α) must be 180° - 115°, or 65°. Since ∠RVS (∠γ) is a vertical angle to ∠QVT, then it is also 65°.

Supplementary Angles - Two angles such as ∠α and ∠β in figure 2, whose measures add up to 180°, or that make a straight angle (straight line), are said to be supplementary. In figure 2, the angles were adjacent to each other, but they don't have to be adjacent to be classified as supplementary angles.

All drawings are in the same plane unless otherwise noted.

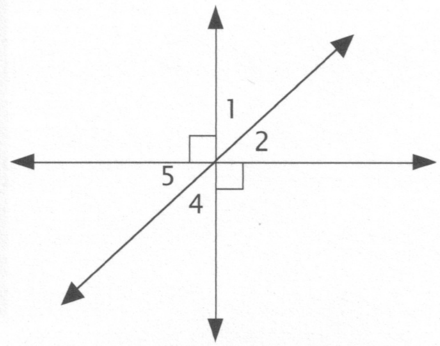
Figure 3



Given: $\overleftrightarrow{DF}$, $\overleftrightarrow{GE}$, and $\overleftrightarrow{KJ}$ all intersect at H.
$\overleftrightarrow{DF} \perp \overleftrightarrow{GE}$

Complementary Angles - We can observe many relationships in figure 3. ∠1 is adjacent to both ∠6 and ∠2. Angle 3 and ∠6 are vertical angles, as are ∠1 and ∠4. Angle 6 and ∠3 are also right angles since $\overleftrightarrow{DF} \perp \overleftrightarrow{GE}$. The new concept here is the relationship between ∠DHE and ∠GHF. Both of these are right angles because the lines are perpendicular; therefore their measures are each 90°. Then m∠1 + m∠2 = 90°, and m∠4 + m∠5 = 90°. Two angles whose measures add up to 90° are called complementary angles. Notice that from what we know about vertical angles, ∠1 and ∠5 are also complementary. Let's use some real measures to verify our conclusions.

Figure 4 (a simplified figure 3)



In figure 4, let's assume that m∠1 = 47°. Then m∠2 must be 43°, since m∠1 and m∠2 add up to 90°. If m∠1 = 47°, then m∠4 must also be 47°, since ∠1 and ∠4 are vertical angles. Also, m∠5 must be 43°. So ∠1 and ∠5 are complementary, as are ∠2 and ∠4. Remember that supplementary and complementary angles do not have to be adjacent to qualify.

It helps me to not get supplementary and complementary angles mixed up if I think of the *s* in straight and the *s* in supplementary. The *c* in complementary may be like the *c* in corner.

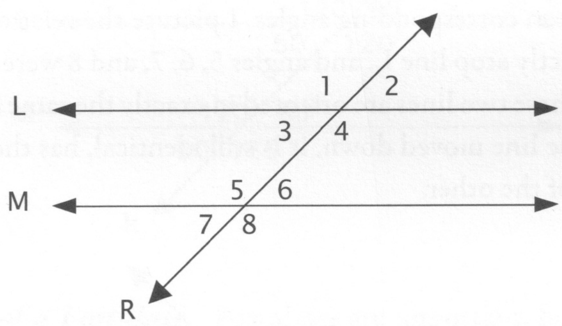
LESSON 7

Transversals

With Interior and Exterior Angles

Transversals - A *transversal* is a straight line that intersects two or more parallel lines. Two parallel lines that lie in the same plane and are cut by a transversal produce several interesting angles.

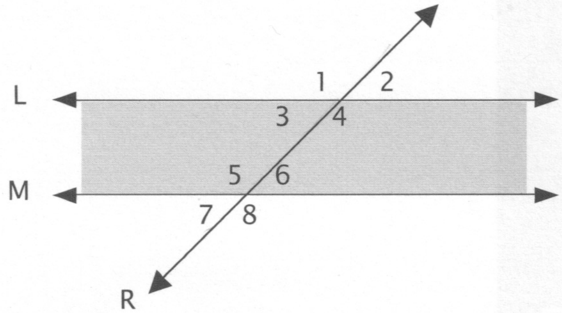
Figure 1



Given: Lines L and M are ||.
Line R intersects each of them and is a transversal.

Interior and Exterior Angles - Angles 3, 4, 5, and 6 are in the inside of the parallel lines and are called *interior* angles. Angles 1, 2, 7, and 8 are on the outside and are called *exterior* angles. See figure 2 on the next page.

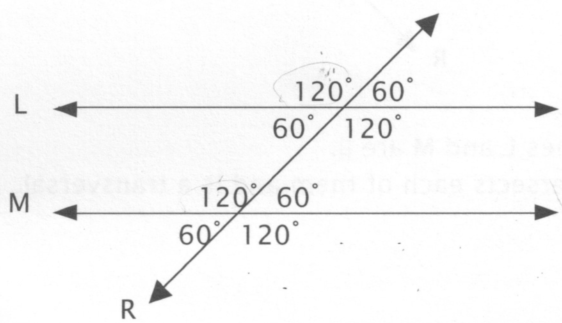
Figure 2



Corresponding Angles - Notice the cluster of four angles around the intersection of lines L and R in figure 2. Also notice the cluster around the intersection of lines M and R. Which angle in the first group *corresponds* to ∠7 in the second group? The answer is ∠3, since both angles are on the lower left-hand side. So ∠1 corresponds to ∠5, ∠2 corresponds to ∠6, and ∠4 corresponds to ∠8.

If we know the measure of ∠1, then we can use what we learned in lesson 6 to find the measures of the other seven angles. Look at figure 3. If m∠1 is 120°, then m∠2 is 60° because they are supplementary, and m∠4 is 120° because it is opposite ∠1 and is a vertical angle. Since m∠2 = 60°, then m∠3 = 60° since they are vertical angles. This much we already know. What this lesson is teaching us is the relationship between corresponding angles. I picture the relationship as if line M was originally directly atop line L, and angles 5, 6, 7, and 8 were covering angles 1, 2, 3, and 4. Since these two lines are oriented in exactly the same direction, they are identical. Since one line moved down, it is still identical, has the same angles, and is an exact image of the other.

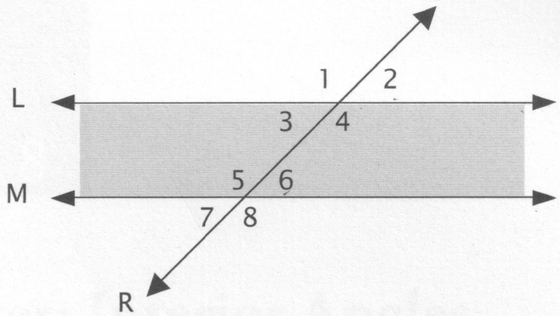
Figure 3



Alternate Angles - Looking at figure 4 on the next page, notice that ∠3 and ∠6 are interior angles, and are on opposite sides of the transversal. They are called *alternate interior* angles. The other pair of alternate interior angles shown are ∠4 and ∠5.

Which do you think are the two pairs of *alternate exterior* angles? One pair is $\angle 1$ and $\angle 8$, and the other pair is $\angle 2$ and $\angle 7$.

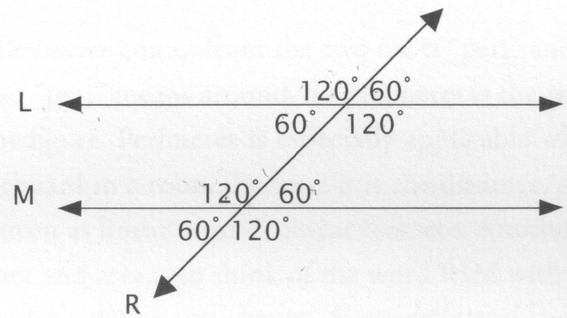
Figure 4



A Postulate - So one of our first observations, or *postulates*, is that when two parallel lines are cut by a transversal, corresponding angles are congruent.

Building on this postulate, it follows that alternate interior angles are congruent, and alternate exterior angles are also congruent. Look at the measurements in figure 5 verify this.

Figure 5



A Converse of a Postulate - Postulates are important, but so are their *converses*, or opposites.

Here is the converse of our first postulate: if two parallel lines cut by a transversal produce congruent corresponding angles, then congruent corresponding angles produce two parallel lines. In figure 4, if it were given that $\angle 3$ and $\angle 7$ are congruent, then we would know that lines L and M are parallel.

LESSON 8

Perimeter; Interior Angles

Rectangle, Triangle, Parallelogram, and Trapezoid

This should be a review lesson for most students, but while going over familiar ground we will also make sure we know the names of and are able to identify these common quadrilaterals: square, rectangle, parallelogram, rhombus, and trapezoid.

Perimeter - *Perimeter* comes from the two roots "peri" and "meter." "Meter" means measure and "peri" means around. So perimeter is the measure of distance around a shape, or figure. Perimeter is especially applicable when measuring for a fence or for baseboard in a room. Because it is the distance, or line around, the answer is always given as linear units or linear feet, etc. Another tip to keep from confusing perimeter and area is to think of the word RIM within perRIMeter.

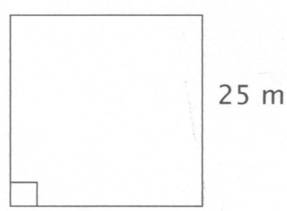
Shapes - Now let's define our shapes. A *quadrilateral* has four sides, as the name reveals: "quad" means four and "lateral" means sides. A *rectangle* is a quadrilateral with four right angles. A *parallelogram* is a quadrilateral with two pairs of parallel sides. A *rhombus* is a quadrilateral with four congruent sides. A *trapezoid* is a quadrilateral with only one pair of parallel sides. A *square* may be defined several ways. It could be a rectangle (four right angles) with four congruent sides, or it could be a rhombus (four congruent sides) with four right angles. It could also be described as a quadrilateral with four congruent sides and four right angles. All of these definitions are correct.

We know that the opposite sides of a parallelogram and of a rectangle have the same lengths. We will prove this formally in lesson 25.

Here are some examples of finding the perimeter. Perimeter is found by adding up the length of all the sides, so ignore any other measurements that may be given.

Example 1

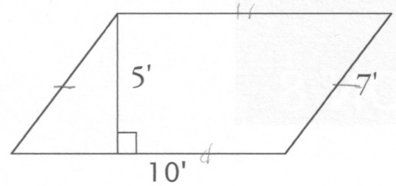
square



P = 25 + 25 + 25 + 25 = 100 m, or
P = 4(25) = 100 m

Example 2

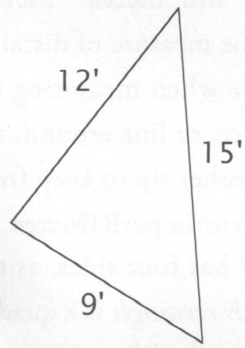
parallelogram



P = 10 + 7 + 10 + 7 = 34 ft

Example 3

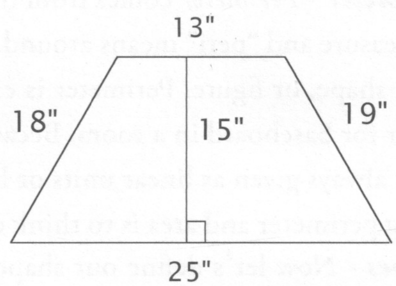
triangle



P = 12 + 15 + 9 = 36 ft

Example 4

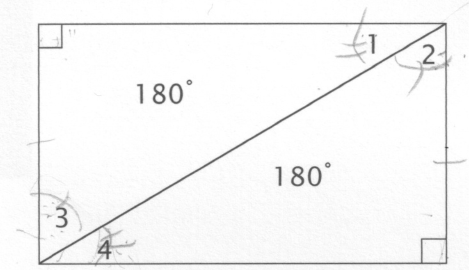
trapezoid



P = 18 + 19 + 13 + 25 = 75 in

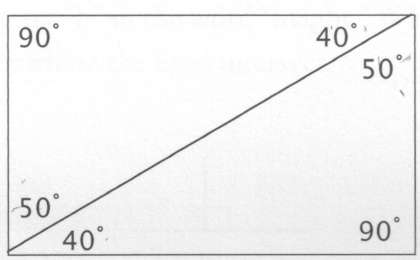
Interior Angles - Notice that if a rectangle has four right angles, then the *sum of the interior angles* is 360°. This is true for all of these quadrilaterals. We deduce this by dividing the quadrilaterals into two triangles by drawing a straight line connecting the opposite vertices.

Figure 1

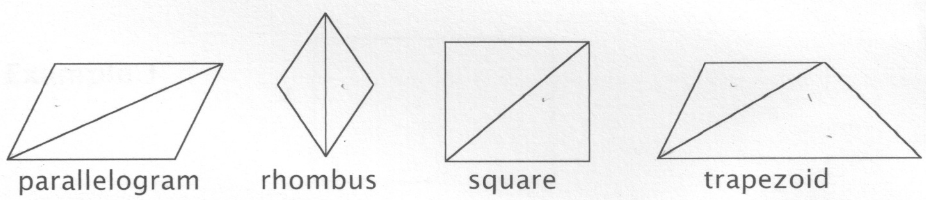


The sum of the angles of a triangle is always 180°. If m∠1 is 40°, then m∠2 must be 50°. Because ∠1 and ∠4 are alternate interior angles, m∠4 is 40° and m∠3, therefore, is 50°.

Figure 2



Adding the angles all up, we see 180° in each triangle and 360° in the quadrilateral. This holds true for all of the following quadrilaterals even if they do not have right angles, because all quadrilaterals can be divided into two triangles with 180° each.

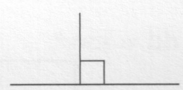


LESSON 9

Area

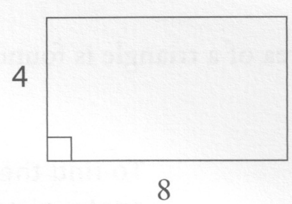
Rectangle, Triangle, Parallelogram, and Trapezoid

Area of a Rectangle - The *area* of a rectangle is found by using the over and the up, or the *base* times the *height*. To remember that the height is always perpendicular, look at the small letter "h" in the word "height." When showing perpendicular lines, we put a square where the lines intersect.



When we do this, we create the letter "h." So the height is always perpendicular to the base. This is very important when finding the area of a triangle, a parallelogram, or a trapezoid. The height is not one of the sloping sides; it is always the perpendicular dimension.

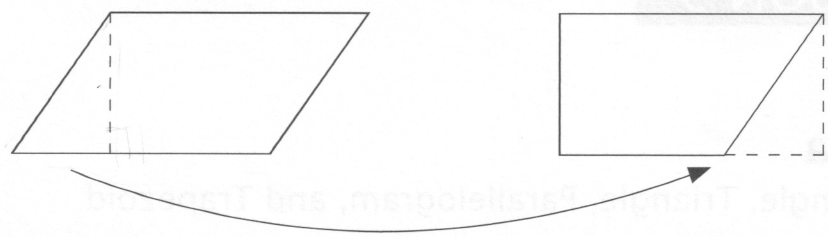
Example 1



Area = bh = 8 x 4 = 32 square units (units^2)

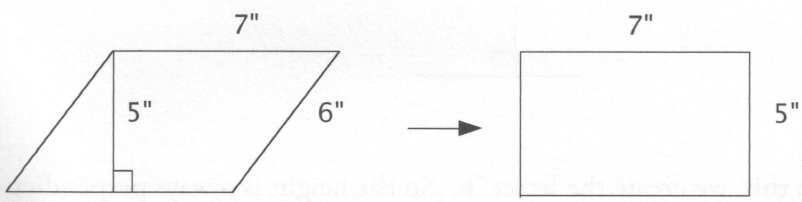
Area of a Parallelogram or Rhombus - To find the area of a parallelogram or rhombus, we use the same formula as we did for finding the area of a rectangle: over times up, or base times height. To understand the formula, look at the following picture. The dotted lines show where we cut off the piece on the left and slid it to the right to make a rectangle. We know the area of a rectangle is base times height.

Figure 1



When finding the area of a parallelogram or trapezoid, remember that the height is always perpendicular to the base. Consider the following example:

Example 2

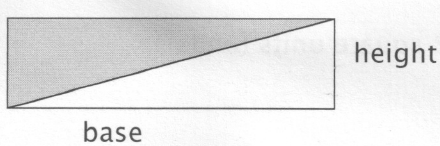


A = bh = 7 x 5 = 35 square inches (in^2)

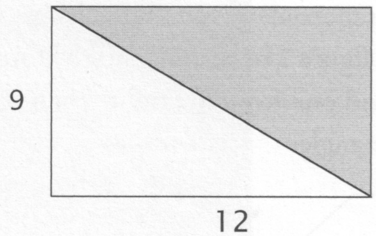
Area is base times height

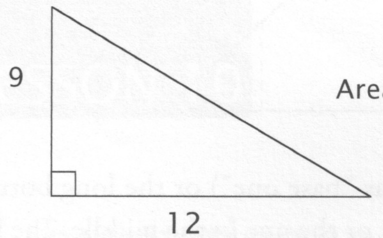
Area of a Triangle - The area of a triangle is found by multiplying the base times the height, times one-half.

Example 3



To find the area of the white triangle, picture it as half of a rectangle. Find the area of the rectangle by multiplying the base times the height. The triangle is half of this, so b x h x 1/2 gives us the area of a triangle.

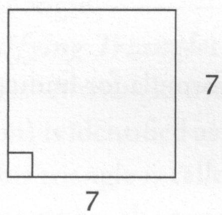




Area = bh x 1/2 = 12 x 9 x 1/2 = 54 units^2

Area of a Square - The area of a square is also found by multiplying the over times the up, or the base times the height. Since the base and the height are the same length in a square, the formula for the area of a square can also be expressed as side times side, or side squared.

Example 4

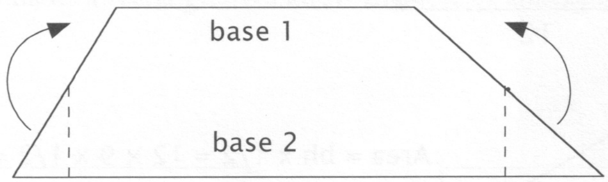


Area = bh = 7 x 7 = 49 units^2
or
Area = SS = S^2 = 7 x 7 = 49 units^2

Notice that when finding area, the answer is always in square units, such as square inches (in^2), square feet (ft^2), etc.

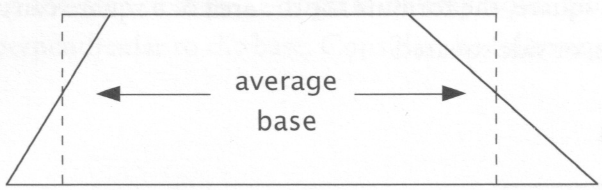
Area of a Trapezoid - To find the area of a trapezoid, we use the same formula as for a rectangle—with a slight change. Look at figure 2 to understand the formula. We cut two corner pieces, one from the left and one from the right. Then pivot them up as if they were on hinges to create a rectangle.

Figure 2



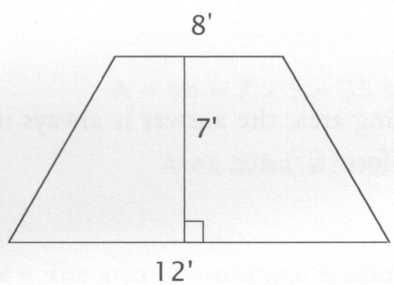
So the base is not the top flat piece (read as "base one") or the long bottom piece (read as "base two"), but the average base, or the one in the middle. The formula is the average base times the height. The formula for finding the area of a trapezoid is traditionally expressed as: $\frac{B_1 + B_2}{2} \times h$

Figure 3



Adding the two bases and dividing by two is the formula for finding the average base.

Example 5



Average base = (8 + 12) ÷ 2 = 10 ft
Area = (avg. b) x h = 10 x 7 = 70 ft^2

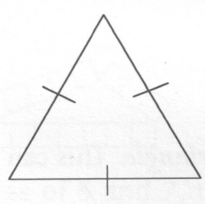
LESSON 10

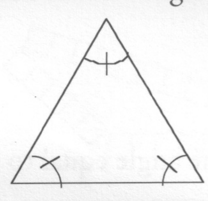
Constructing and Identifying Triangles

Constructing a Triangle - A triangle consists of three angles and three sides. Construct a triangle that is large enough so that you can measure each angle with your protractor. Measure each angle and each side. Do you notice a relationship between an angle and the side opposite the angle? The smaller the angle is, the smaller the side opposite that angle. The larger the angle is, the larger the side opposite that angle.

Identifying Triangles - Triangles may be described in terms of their sides or their angles. A triangle with all sides congruent and all angles congruent (regular polygon) is identified as *equilateral* (equal sides) or *equiangular* (equal angles). Usually this triangle is called equilateral. Since the interior angles add up to 180° and each angle has the same measure, the measure of each angle is 60°.

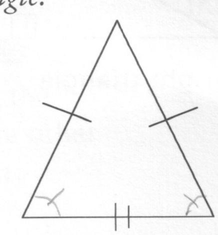
Figure 1





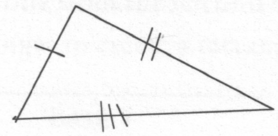
We use the slash marks to show congruent sides. A triangle with two sides congruent is called an *isosceles triangle.*

Figure 2



A triangle with no sides congruent is called a *scalene triangle.*

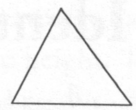
Figure 3



scalene

A triangle with all three angles greater than 0°, and each less than 90°, is an *acute triangle.* This can be expressed as: each angle > 0° and < 90°, or 0° < each angle < 90°.

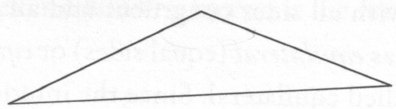
Figure 4



acute triangle

A triangle with one angle greater than 90° is an *obtuse triangle.* This can be expressed as: one angle > 90° and <180°, or 90° < one angle < 180°.

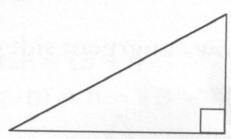
Figure 5



obtuse triangle

A triangle with one angle equal to 90° is a *right triangle.* This can be expressed as: one angle = 90°.

Figure 6

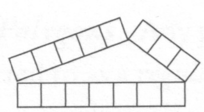


right triangle

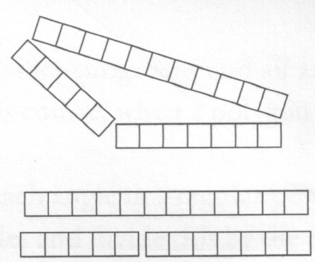
Could there be an acute triangle that is also equilateral? Yes, because if it is equilateral, then all three angles have a measure of 60°, which means they are all acute as well. Could there be an isosceles right triangle? Yes, if the legs were congruent.

Limitations - We know that there are limitations in the measures of the angles in a triangle. If one angle is 80°, and another is 70°, then the third angle must be 30°. There are also limitations on the lengths of the sides of a triangle. I'd like you to figure this one out yourself. Construct a triangle using three of the unit bars: the orange two bar, the light blue five bar, and the tan seven bar. Can you do it? You can see that you will need at least the pink three bar in place of the two bar. If you have just the five bar and the seven bar, you can also see that the third bar can't be any longer than twelve, and using whole numbers, must be at the most eleven units long. Try two five bars with the nine bar. Can you do it now? Can you develop a principle based on these observations? The two smaller sides together must be longer than the third side.

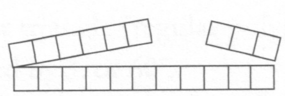
Notice how the following expression represents this observation: if side A ≤ side B ≤ side C (so A is smaller or equal to side B, and C is the longest), then A + B > C (the two smaller sides added up must be longer than C). Looking at the lengths of the sides of another triangle, 6 in–7 in–11 in, could these be the dimensions of a triangle? Yes, because the sum of the two smaller sides, 6 in + 7 in, is longer than 11 in, or 6 in + 7 in > 11 in. What about a triangle with sides of 6 cm–3 cm–11 cm? No, these could not be the dimensions of a triangle, because 3 + 6 < 11.



With sides of 5 and 7, the small side must be > 2.



With sides of 5 and 7, the third side must be < 12.



With the long side 11, the combined length of the other two sides can't be < 11.