

16. $\angle FGE \cong \angle BGC$: vertical angles
 $m\angle BGC + m\angle AGB = 90^\circ$:
 complementary angles
 $m\angle AGB = 90^\circ - 43^\circ = 47^\circ$
17. $m\angle EGD = 90^\circ - 43^\circ = 47^\circ$:
 complementary angles
 $m\angle EGC = 90^\circ + 47^\circ = 137^\circ$
18. $m\angle BGC = m\angle FGC - m\angle FGB =$
 $180^\circ - 135^\circ = 45^\circ$
19. $m\angle AGB = m\angle FGB - 90^\circ =$
 $135^\circ - 90^\circ = 45^\circ$
 $m\angle EGD = m\angle AGB = 45^\circ$:
 vertical angles
20. semicircle:
 $C = \frac{1}{2}(2\pi r) \approx$
 $\frac{1}{2}(2)(3.14)(3.3) = 10.36$ ft
 sides of triangle:
 $6.2 + 6.2 = 12.4$ ft
 total:
 $10.36 + 12.4 = 22.76$ ft

Systematic Review 13E

- diameter
- $A = \pi r^2$
- $\frac{22}{7}$
- 3.14
- latitude
- longitude
- $A = \frac{1}{2}(9) \times \frac{1}{2}(3) \times \pi \approx$
 $(4.5)(1.5)(3.14) = 21.2$ in²
- 60
- sphere
- diameter
- $A = \pi r^2 \approx (3.14)(3.5)^2 = 38.47$ m²
- $C = \pi d \approx (3.14)(1.25) = 3.925$ or 3.93 cm
- $360^\circ \div 40^\circ = 9$
- infinite

- exterior
- alternate exterior
- MLK, CEH
- GHE

$$19. \quad \begin{array}{rcl} 3Y + 2X & = & 12 \\ (2)(4Y - X = 5) & \Rightarrow & 8Y - 2X = 10 \\ \hline 11Y & = & 22 \\ Y & = & 2 \end{array}$$

$$3Y + 2X = 12 \Rightarrow 3(2) + 2X = 12$$

$$6 + 2X = 12$$

$$2X = 6$$

$$X = 3$$

solution = (3, 2)

$$20. \quad \begin{array}{rcl} Y - X & = & -3 \\ Y - 2X = -4 & \Rightarrow & -Y + 2X = 4 \\ \hline X & = & 1 \end{array}$$

$$Y - X = 3 \Rightarrow Y - (1) = -3$$

$$Y = -2$$

solution = (1, -2)

Lesson Practice 14A

- base, height
- faces
- squares
- edges
- circle
- cubic
- vertices
- $V = (4)(4)(4) = 64$ in³
- $V = (5)(4)(3) = 60$ ft³
- $V = Bh = \pi r^2 h \approx$
 $(3.14)(10^2)(5) = 1,570$ ft³
- $V = (5)(7)(4) = 140$ ft³
- $V = Bh = \pi r^2 h \approx$
 $(3.14)(10^2)(25) = 7,850$ ft³

Lesson Practice 14B

1. g
2. a
3. c
4. b
5. d
6. h
7. f
8. e

$$9. V = (20)(30)(10) = 6,000 \text{ ft}^3$$

$$10. V = Bh = \pi r^2 h \approx (3.14)(15^2)(60) = 42,390 \text{ ft}^3$$

$$11. V = (10)(10)(10) = 1,000 \text{ ft}^3$$

$$12. V = Bh = \pi r^2 h \approx (3.14)(10^2)(15) = 4,710 \text{ ft}^3$$

Systematic Review 14C

$$1. V = (3)(5)(4) = 60 \text{ m}^3$$

$$2. V = Bh = \pi r^2 h \approx (3.14)(4^2)(4) = 200.96 \text{ ft}^3$$

$$3. V = Bh = \pi r^2 h \approx (3.14)(6^2)(10) = 1,130.4 \text{ ft}^3$$

$$4. A = (8)(8) = 64 \text{ cm}^2$$

$$5. A = \pi r^2 \approx (3.14)(6^2) = 113.04 \text{ cm}^2$$

$$6. A = (3)(5)(\pi) \approx (3)(5)(3.14) = 47.1 \text{ cm}^2$$

$$7. (N-2)180^\circ \Rightarrow ((8)-2)180^\circ = (6)(180^\circ) = 1,080^\circ \text{ total}$$

$$1,080^\circ \div 8 = 135^\circ \text{ per angle}$$

$$8. 180^\circ - 135^\circ = 45^\circ \text{ or } 360^\circ \div 8 = 45^\circ; 45^\circ(8) = 360^\circ$$

$$9. 360^\circ \div 10^\circ = 36 \text{ sides}$$

10. sphere

$$11. C = 2\pi r \approx (2)(3.14)(6) = 37.68 \text{ in}$$

12. 0°

13. point

$$14. A = \frac{5+7}{2}(13) = 78 \text{ in}^2$$

$$15. 180^\circ - (65^\circ + 15^\circ) = 180^\circ - 80^\circ = 100^\circ$$

16. obtuse

$$17. m\angle CLM = m\angle EHL = 115^\circ:$$

corresponding angles

$$m\angle MLK = 180^\circ - m\angle EHL = 180^\circ - 115^\circ = 65^\circ$$

supplementary angles

$$18. m\angle ACL = m\angle MLK = 65^\circ:$$

corresponding angles

$$19. \angle CED = m\angle EHL = 115^\circ:$$

corresponding angles

$$20. m\angle EHG = 180^\circ - m\angle EHL = 180^\circ - 115^\circ = 65^\circ:$$

supplementary angles

Systematic Review 14D

$$1. V = (7)(10)(8) = 560 \text{ m}^3$$

$$2. V = Bh = \pi r^2 h \approx (3.14)(3.5^2)(3) = 115.40 \text{ ft}^3$$

$$3. V = Bh = \pi r^2 h \approx (3.14)(4^2)(7) = 351.68 \text{ in}^3$$

$$4. A = (5.4)(4.1) = 22.14 \text{ cm}^2$$

$$5. A = \pi r^2 \approx (3.14)(11^2) = 379.94 \text{ cm}^2$$

$$6. V = (10)(4)(\pi) \approx (10)(4)(3.14) = 125.6 \text{ cm}^2$$

$$7. (N-2)180^\circ \Rightarrow ((10)-2)180^\circ = (8)(180^\circ) = 1,440^\circ$$

$$1,440^\circ \div 10 = 144^\circ$$

$$8. 180^\circ - 144^\circ = 36^\circ$$

or $360^\circ \div 10 = 36^\circ;$

$$36^\circ \times 10 = 360^\circ$$

$$9. 360^\circ \div 45^\circ = 8$$

10. diameter

11. 23°

12. 0°

13. plane

14. yes, because $3 + 4 > 5$

15. $m\angle 10 = m\angle BCA = 62^\circ$
definition of bisector
16. $m\angle 10 + m\angle 11 = 62^\circ + 62^\circ = 124^\circ$
17. $m\angle 9 = 180^\circ - (m\angle 10 + m\angle 11) =$
 $180^\circ - (124^\circ) = 56^\circ$
supplementary angles
18. $m\angle 9 = 56^\circ$ (from problem 17)
 $m\angle 1 = 56^\circ$: corresponding angles
19. 62° : $m\angle 7 = m\angle BCA$ because they
are alternate interior angles.
20. $m\angle 11 = m\angle 12$: vertical angles
 $m\angle 7 = m\angle 11 = 58^\circ$:
alternate interior
or:
 $m\angle 7 = m\angle 12 = 58^\circ$: corresponding

Systematic Review 14E

1. $V = (20)(30)(10) = 6,000 \text{ m}^3$
2. $V = Bh = \pi r^2 h \approx$
 $(3.14)(10^2)(15) = 4,710 \text{ ft}^3$
3. $V = Bh = \pi r^2 h \approx$
 $(3.14)(5^2)(7) = 549.5 \text{ in}^3$
4. $A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24 \text{ in}^2$
5. $A = \pi r^2 \approx (3.14)(4^2) = 50.24 \text{ cm}^2$
6. $A = \frac{1}{2}(20) \times \frac{1}{2}(15) \times \pi \approx$
 $(10)(7.5)(3.14) = 235.5 \text{ units}^2$
7. $(N-2)180^\circ \quad ((6)-2)180^\circ =$
 $(4)180^\circ = 720^\circ$
 $720^\circ \div 6 = 120^\circ$
8. $180^\circ - 120^\circ = 60^\circ$ or $360^\circ \div 6 = 60^\circ$
 360° is always the total
of exterior degrees.
9. $360^\circ \div 60^\circ = 6$ sides
10. tangent
11. $C = 2\pi r \approx (2)(3.14)(15) = 94.2 \text{ in}$
12. $60'$
13. line (or line segment or ray)
14. $P = 25 + 25 + 25 + 25 = 100 \text{ ft}$

15. $180^\circ - (23^\circ + 35^\circ) =$
 $180^\circ - 58^\circ = 122^\circ$
16. minor arc = 23° ;
major arc = $360^\circ - 23^\circ = 337^\circ$
17. $\sqrt{16} = 4$
18. $\sqrt{100} = 10$
19. $\sqrt{25} = 5$
20. $\sqrt{144} = 12$

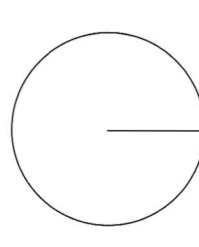
Lesson Practice 15A

1. slant height
2. altitude
3. vertex
4. $\frac{1}{3}$
5. circle
6. congruent, parallel
7. parallelograms
8. $\frac{4}{3}\pi r^3$
9. $V = \frac{1}{3}Bh = \frac{1}{3}(5)(5)(6) = 50 \text{ in}^3$
10. $V = \frac{1}{3}Bh \approx$
 $\frac{1}{3}(3.14)(3^2)(11) = 103.62 \text{ in}^3$
11. $V = Bh = \frac{1}{2}(8)(9)(15) = 540 \text{ ft}^3$
12. $V = \frac{4}{3}\pi r^3 \approx$
 $\frac{4}{3}(3.14)(2^3) = 33.49 \text{ ft}^3$ In this
and in other problems of this
type, you may ignore small
answer variations caused by
differences in rounding technique.
13. $V = \frac{1}{3}Bh = \frac{1}{3}(10)(10)(40)$
 $\approx 1,333.33 \text{ in}^3$
14. $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h \approx$
 $\frac{1}{3}(3.14)(2^2)(6) = 25.12 \text{ in}^3$

Lesson Practice 15B

- prism
- volume; sphere
- base; height
- $\frac{1}{3}$
- square
- altitude
- $\frac{1}{3}$
- face
- $V = \frac{1}{3}Bh = \frac{1}{3}(3.6)(3.6)(4)$
 $= 17.28 \text{ ft}^3$
- $V = \frac{1}{3}Bh \approx$
 $\frac{1}{3}(3.14)(4.2^2)(9.7) = 179.09 \text{ ft}^3$
- $V = Bh = \frac{1}{2}(4.8)(3.2)(7.8)$
 $= 59.9 \text{ cm}^3$
- $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3.14)(1^3) = 4.19 \text{ ft}^3$
- pyramid:
 $V = \frac{1}{3}Bh = \frac{1}{3}(6)(6)(3.6) = 43.2 \text{ ft}^3$
rectangular solid:
 $V = (6)(6)(2.7) = 97.2 \text{ ft}^3$
total:
 $V = 43.2 + 97.2 = 140.4 \text{ ft}^3$
- cone: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h \approx$
 $\frac{1}{3}(3.14)(6)^2(14) = 527.52 \text{ in}^3$
cylinder: $V = Bh =$
 $\pi r^2 h \approx (3.14)(6^2)(11) = 1,243.44 \text{ in}^3$
total: $V = 527.52 + 1,243.44$
 $= 1,770.96 \text{ in}^3$

- $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 \approx$
 $\frac{1}{3}(3.14)(4.5^2)(8.25) = 174.86 \text{ in}^3$
- prism:
 $V = Bh = \frac{1}{2}(4)(5)(6) = 60 \text{ ft}^3$
rectangular solid:
 $V = (5)(6)(2) = 60 \text{ ft}^3$
total:
 $V = 60 + 60 = 120 \text{ ft}^3$
- $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3.14)(4^3)$
 $= 267.95 \text{ in}^3$
- altitude
- They are parallel. (or congruent)
- $V = (20)(20)(20) = 8,000 \text{ in}^3$
- $A = \pi r^2 \approx (3.14)(5^2) = 78.5 \text{ ft}^2$
- $A = \frac{1}{2}(2) \times \frac{1}{2}(10) \times \pi \approx (1)(5)(3.14)$
 $= 15.7 \text{ ft}^2$
- circumference
- $(N-2)180^\circ \quad ((10)-2)180^\circ =$
 $(8)180^\circ = 1,440^\circ$
 $1,440^\circ \div 10 = 144^\circ$
- If interior angle is 120° ,
then exterior angles are
 $180^\circ - 120^\circ$ or 60° .
Exterior angles always add up
to 360° . $360^\circ \div 60^\circ = 6$ sides
- use protractor to check
- use protractor to check: New
angles should each measure 62.5° .
- A radius and tangent that touch
a circle at the same point are
always perpendicular to each other.

**Systematic Review 15C**

- $V = \frac{1}{3}Bh = \pi \times \frac{1}{3} \times \frac{5}{2} \times \frac{5}{2} \times \frac{9}{2} =$
 $\frac{225}{24} = 9\frac{3}{8} \text{ m}^3$ or
 $V = \frac{1}{3}Bh = \frac{1}{3}(2.5)(2.5)(4.5) = 9.375 \text{ m}^3$

16. $m\angle 6 = m\angle 14 = 50^\circ$:
 corresponding angles
 $m\angle 5 = 180^\circ - m\angle 6 =$
 $180^\circ - 50^\circ = 130^\circ$:
 supplementary angles
 There are other valid ways of
 arriving at this answer.
17. $m\angle BCD = 180^\circ - m\angle 14 =$
 $180^\circ - 50^\circ = 130^\circ$:
 supplementary angles
 $m\angle 10 = \frac{1}{2} m\angle BCD = \frac{1}{2} (130^\circ) = 65^\circ$:
 line AC bisects $\angle BCD$
18. $m\angle 11 = 65^\circ$ from last problem
 $m\angle 4 = m\angle 11 = 65^\circ$:
 corresponding angles
 $m\angle 8 = 180^\circ - m\angle 4 =$
 $180^\circ - 65^\circ = 115^\circ$:
 supplementary angles
19. $m\angle 11 = 65^\circ$ from problem 17
 $m\angle 12 = m\angle 11 = 65^\circ$:
 vertical angles
20. complementary; If two angles are
 complementary, they add up to 90° .

Systematic Review 15D

1. $V = \frac{1}{3} Bh =$
 $\frac{1}{3} \times \frac{13}{2} \times \frac{13}{2} \times \frac{7}{2} = \frac{1,183}{24} = 49\frac{7}{24} \text{ m}^3$
 or
 $\frac{1}{3} (6.5)(6.5)(3.5) \approx 49.29 \text{ m}^3$
2. $V = \frac{1}{3} Bh = \frac{1}{3} \pi r^2 h \approx$
 $\frac{1}{3} (3.14)(3.4^2)(7.6) = 91.96 \text{ in}^3$

3. cone:
 $V = \frac{1}{3} Bh = \frac{1}{3} \pi r^2 h \approx$
 $\frac{1}{3} (3.14)(4^2)(12) = 200.96 \text{ in}^3$
 cylinder:
 $V = Bh = \pi r^2 h \approx (3.14)(4^2)(10) = 502.4 \text{ in}^3$
 total:
 $V = 200.96 + 502.4 = 703.36 \text{ in}^3$
4. $V = \frac{4}{3} \pi r^3 \approx \frac{4}{3} (3.14)(2^3) = 33.49 \text{ in}^3$
5. parallelogram
6. chord
7. $A = bh = (20)(15) = 300 \text{ in}^2$
8. $P = 20 + 20 + 20 + 20 = 80 \text{ in}$
9. $A = \frac{1}{2} (3) \times \frac{1}{2} (6) \times \pi \approx$
 $(1.5)(3)(3.14) = 14.13 \text{ ft}^2$
10. scalene
11. $(N - 2)180^\circ \Rightarrow ((8) - 2)180^\circ =$
 $(6)180^\circ = 1,080^\circ$
 $1080^\circ \div 8 = 135^\circ$
12. exterior angles $= 180^\circ - 150^\circ = 30^\circ$
 $360^\circ \div 30^\circ = 12$ sides
13. check with ruler
14. check with ruler:
 each half should be $2\frac{3}{8} \text{ in}$
15. check with protractor
16. midpoint
17. $AF = \frac{1}{2} AD = \frac{1}{2} \times 2\frac{7}{8} = 1\frac{7}{16}$
18. $m\angle BFD = m\angle AFE = 88.5^\circ$:
 vertical angles
 $m\angle BFC = \frac{1}{2} m\angle BFD = \frac{1}{2} (88.5^\circ) = 44.25^\circ$
 bisector
19. $m\angle EFD = 180^\circ - m\angle AFE =$
 $180^\circ - 88.5^\circ = 91.5^\circ$:
 supplementary angles
20. supplementary; If two angles add
 up to 180° , they are supplementary.

Systematic Review 15E

1. $V = Bh = \frac{1}{2}(4.5)(4.2)(11)$
 $= 103.95 \text{ mm}^3$
2. $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3.14)(1.5^3)$
 $= 14.13 \text{ cm}^3$
3. pyramid:
 $V = \frac{1}{3}Bh = \frac{1}{3}(8)(8)(6) = 128 \text{ ft}^3$
 rectangular solid:
 $V = (8)(8)(3) = 192 \text{ ft}^3$
 total:
 $V = 128 + 192 = 320 \text{ ft}^3$
4. altitude
5. prism
6. $\frac{1}{3}$
7. $A = bh = (8)(3.5) = 28 \text{ in}^2$
8. $P = 8 + 4 + 8 + 4 = 24 \text{ in}$
9. $A = \frac{1}{2}(8) \times \frac{1}{2}(10) \times \pi \approx (4)(5)(3.14)$
 $= 62.8 \text{ ft}^2$
10. equilateral
11. $(N-2)180^\circ \Rightarrow ((5)-2)180^\circ =$
 $(3)180^\circ = 540^\circ$
 $540^\circ \div 5 = 108^\circ$
12. 8
13. exterior angles =
 $180^\circ - 135^\circ = 45^\circ$;
 $360^\circ \div 45^\circ = 8 \text{ sides}$
14. check with ruler
15. check with protractor:
 angles should measure 55°
16. check with protractor
17. $8^3 \cdot 8^4 = 8^{3+4} = 8^7$
18. $2^8 \div 2^3 = 2^{8-3} = 2^5$
19. $X^2X^3Y^5Y^{-1} = X^{2+3}Y^{5+(-1)} = X^5Y^4$
20. $\frac{1}{10^3} = \frac{10^{-3}}{1} = 10^{-3}$

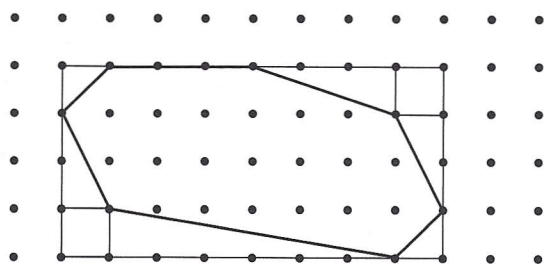
Lesson Practice 16A

1. 6
2. 5
3. 4
4. circles; rectangle
5. square
6. height; circumference
7. $SA = 2(3)(5) + 2(3)(4) + 2(4)(5) =$
 $30 + 24 + 40 = 94 \text{ ft}^2$
8. $SA = 2\pi r^2 + 2\pi rh \approx$
 $(2)(3.14)(10^2) + (2)(3.14)(10)(5) =$
 $628 + 314 = 942 \text{ ft}^2$
9. $SA = (5)(5) + (4) \frac{1}{2} (5)(6) =$
 $25 + 60 = 85 \text{ in}^2$
10. base:
 $SA = \frac{1}{2}(6)(5) = 15 \text{ ft}^2$
 sides:
 $SA = (3) \frac{1}{2} (6)(8) = 72 \text{ ft}^2$
 total:
 $SA = 15 + 72 = 87 \text{ ft}^2$
11. $SA = 2(5)(4) + 2(5)(7) + 2(4)(7) =$
 $40 + 70 + 56 = 166 \text{ ft}^2$
12. "roof":
 $SA = 2(4.5)(6) = 54 \text{ ft}^2$
 sides:
 $SA = 2(4)(5) + 2(4)(6)$
 $= 40 + 48 = 88 \text{ ft}^2$
 triangles:
 $SA = 2 \frac{1}{2} (4)(5) = 20 \text{ ft}^2$
 bottom:
 $SA = (5)(6) = 30 \text{ ft}^2$
 total:
 $SA = 54 + 88 + 20 + 30 = 192 \text{ ft}^2$

Lesson Practice 16B

1. square
2. pyramid
3. cube

6. See illustration



7. $4 \times 8 = 32 \text{ units}^2$

8. $1 \times 2 = 2 \text{ units}^2$

$$\frac{1}{2}(1 \times 3) = 1.5 \text{ units}^2$$

$$\frac{1}{2}(2 \times 3) = 3 \text{ units}^2$$

$$\frac{1}{2}(2 \times 2) = 2 \text{ units}^2$$

$$\frac{1}{2}(1 \times 7) = 3.5 \text{ units}^2$$

$$\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$$

9. $2 + 1.5 + 3 + 2 + 3.5 + 1 = 13 \text{ units}^2$

$$32 - 13 = 19 \text{ units}^2$$

10. $10 \times 5 = 50 \text{ units}^2$

$$\frac{1}{2}(5 \times 2) = 5 \text{ units}^2$$

$$1 \times 5 = 5 \text{ units}^2$$

$$\frac{1}{2}(1 \times 5) = 2.5 \text{ units}^2$$

$$\frac{1}{2}(4 \times 1) = 2 \text{ units}^2$$

$$1 \times 1 = 1 \text{ unit}^2$$

$$\frac{1}{2}(6 \times 1) = 3 \text{ units}^2$$

$$\frac{1}{2}(3 \times 4) = 6 \text{ units}^2$$

$$5 + 5 + 2.5 + 2 + 1 + 3 + 6 = 24.5 \text{ units}^2$$

$$50 - 24.5 = 25.5 \text{ units}^2$$

11. $4 \times 8 = 32 \text{ units}^2$

$$\frac{1}{2}(1 \times 1) = .5 \text{ units}^2$$

$$\frac{1}{2}(3 \times 1) = 1.5 \text{ units}^2$$

$$1 \times 1 = 1 \text{ unit}^2$$

$$\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$$

$$\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$$

$$1 \times 1 = 1 \text{ unit}^2$$

$$\frac{1}{2}(5 \times 1) = 2.5 \text{ units}^2$$

$$\frac{1}{2}(2 \times 1) = 1 \text{ unit}^2$$

$$.5 + 1.5 + 1 + 1 + 1 + 1 + 1 + 2.5 + 1$$

$$= 9.5 \text{ units}^2$$

$$32 - 9.5 = 22.5 \text{ units}^2$$

Honors Lesson 14

1. $\frac{1}{2}(3 \times 4) = \frac{1}{2}(12) = 6 \text{ units}^2$

2. $A = \sqrt{s(s-a)(s-b)(s-c)}$

$$A = \sqrt{6(6-3)(6-4)(6-5)}$$

$$A = \sqrt{6(3)(2)(1)}$$

$$A = \sqrt{36}$$

$$A = 6 \text{ units}^2$$

yes

$$3. A = \sqrt{16(16-7)(16-10)(16-15)}$$

$$A = \sqrt{16(9)(6)(1)}$$

$$A = \sqrt{864}$$

$$A = 29.39 \text{ units}^2$$

$$4. A = \sqrt{52(52-36)(52-28)(52-40)}$$

$$A = \sqrt{52(16)(24)(12)}$$

$$A = \sqrt{239,616}$$

$$A = 489.51 \text{ units}^2$$

$$5. V = \pi r^2 h$$

$$V = 3.14(2)^2(10)$$

$$V = 3.14(4)(10)$$

$$V = 125.6 \text{ in}^3$$

$$6. V = \pi r^2 h$$

$$V = 3.14(1)^2(10)$$

$$V = 3.14(1)(10)$$

$$V = 31.4 \text{ in}^3$$

It is $\frac{1}{4}$ the first one

$$7. V = \pi r^2 h$$

$$V = 3.14(2)^2(5)$$

$$V = 3.14(4)(5)$$

$$V = 62.8 \text{ in}^3$$

It is half the first one.

$$8. V = \pi r^2 h$$

$$V = 3.14(4)^2(10)$$

$$V = 3.14(16)(10)$$

$$V = 502.4 \text{ in}^3$$

It is four times the first one.

$$9. V = \pi r^2 h$$

$$V = 3.14(2)^2(20)$$

$$V = 3.14(4)(20)$$

$$V = 251.2 \text{ cu in}^3$$

It is two times the first one

10. When the height is doubled, the volume is doubled. When the height is halved, the volume is halved. When the radius is doubled, the volume increases by a factor of 4. When the radius is halved, the volume decreases by a factor of 4. The student may use his own words to express this.

11. Answers will vary.

12. Take the formula, and multiply both sides by 2:

$$V = \pi r^2 h$$

$$2V = 2\pi r^2 h$$

Now rearrange the factors:

$$V = \pi r^2 h$$

$$2V = \pi r^2 h$$

Take the formula, and multiply both sides by 4:

$$V = \pi r^2 h$$

$$4V = 4\pi r^2 h$$

Rewrite the 4 on the right side as 2^2 :

$$4V = 2^2 \pi r^2 h$$

Rearrange the factors:

$$4V = \pi 2^2 r^2 h$$

$$4V = \pi (2r)^2 h$$

There is more than one way to set this up. As long as you show the same results as by experimentation, the answer is correct.

Honors Lesson 15

1. $3 \times 3 \times 3 = 27 \text{ ft}$
2. $12 \times 12 \times 12 = 1,728 \text{ in}^3$
3. $8 \times 4 \times 2 = 64 \text{ in}^3$
 $64 \times .3 = 19.2 \text{ lbs}$
4. $64 \text{ in}^3 \div 1,728 = .037 \text{ ft}^3$
 $.037 \times 1200 = 44.4 \text{ lbs}$
 You could probably lift it,
 but it would be much heavier
 than expected.
5. First find what the volume would
 be if it were solid:
 $V = \pi r^2 h$
 $V = 3.14(.5)^2(12)$
 $V = 9.42 \text{ in}^3$
 Now find the volume inside
 the pipe:
 $V = \pi r^2 h$
 $V = 3.14(.25)^2(12)$
 $V = 2.355 \text{ in}^3$
 Then find the difference:
 $9.42 - 2.355 = 7.065 \text{ in}^3$
6. $7.065 \times .26 = 1.8369 \text{ lbs}$

7. $V = \frac{4}{3} \pi r^3$
 $V = \frac{4}{3} (3.14)(.25)^3$
 $V = .07 \text{ in}^3 (\text{rounded})$
 $.07 \times .3 \approx .002 \text{ pounds for}$
 one bearing
 $25 \div .02 = 12,500 \text{ bearings}$
 Because we rounded some
 numbers, the actual number
 of bearings in the box may be
 slightly different. Keep in mind
 that the starting weight was
 rounded to a whole number.
 Our answer is close enough to
 be helpful in a real life situation,
 where someone wants to know
 approximately how many bearings
 are available without counting.
8. The side view is a trapezoid,
 and the volume of the water
 is the area of the trapezoid
 times the width of the pool:
 $A = \frac{3+10}{2} (40)$
 $A = 6.5(40)$
 $A = 260 \text{ ft}^2$
 $V = 260(20)$
 $V = 5,200 \text{ ft}^3$
9. Volume of the sphere:
 $V = \frac{4}{3} (3.14)(1)^3 \text{ units}^3$
 $V = 4.19$
 Volume of the cube:
 $V = 2 \times 2 \times 2 = 8 \text{ units}^3$
 $8 - 4.19 = 3.81 \text{ units}^3$

10. Volume of the cylinder:

$$V = 3.14 (1)^2 (2)$$

$$V = 6.28 \text{ units}^3$$

Volume of the sphere from #9:

$$4.19 \text{ units}^3$$

$$6.28 - 4.19 = 2.09 \text{ units}^3$$

Note: You may use the fractional value of π if it seems more convenient.

Honors Lesson 16

- $(r)\pi r = \pi r^2$
- $$A = LW + LW + LH + LH + WH + WH$$

$$= 2LW + 2LH + 2WH$$

$$= 2(LW + LH + WH)$$
- $2(s^2 + s^2 + s^2) = 2(3s^2) = 6s^2$
- $$V = 3(11)(3) = 99 \text{ ft}^3$$

$$SA = 2(3 \times 11) + 2(3 \times 3) + 2(11 \times 3)$$

$$= 2(33) + 2(9) + 2(33)$$

$$= 66 + 18 + 66$$

$$= 150 \text{ ft}^2$$
- $$150 \text{ ft}^2 \div 6 \text{ faces} = 25 \text{ ft}^2 \text{ per face}$$

$$\sqrt{25} = 5 \text{ ft}$$

The new bin is $5 \times 5 \times 5$.
- The cube-shaped one holds more.
 $125 - 99 = 26 \text{ ft}^3$ difference.

Honors Lesson 17

- $$V = \pi r^2 h$$

$$V = 3.14 (2)^2 (4)$$

$$V = 50.24 \text{ ft}^3$$

$$2. \quad V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} (3.14) (2)^3$$

$$V = 33.49 \text{ ft}^3 \text{ (rounded)}$$

$$3. \quad V = 3.14 (3)^2 (6)$$

$$V = 169.56 \text{ units}^3$$

$$4. \quad V = \frac{4}{3} (3.14) (3)^3$$

$$V = 113.04 \text{ units}^3 \text{ (rounded)}$$

$$5. \quad V = 3.14 (1)^2 (2)$$

$$V = 6.28 \text{ units}^3$$

$$6. \quad V = \frac{4}{3} (3.14) (1)^3$$

$$V = 4.19 \text{ units}^3 \text{ (rounded)}$$

$$7. \quad \frac{33.49}{50.24} \approx .67 \quad \frac{113.04}{169.56} \approx .67$$

$$\frac{4.19}{6.28} \approx .67$$

$$8. \quad \frac{2}{3}$$

$$9. \quad A = 2\pi r^2 + 2\pi rh$$

$$A = 2(3.14)(3)^2 + 2(3.14)(3)(6)$$

$$A = 56.52 + 113.04 = 169.56 \text{ units}^2$$

$$10. \quad A = 4(3.14)(3)^2$$

$$A = 113.04 \text{ units}^2$$

$$11. \quad \frac{113.04}{169.56} \approx \frac{2}{3}$$

- The surface area and volume of a sphere appear to be $\frac{2}{3}$ of the surface area and volume of a cylinder with the same dimensions. (Archimedes proved that this is the case.)

Honors Lesson 18

- 4,003 mi
- 90° ; a tangent to a circle is perpendicular to the diameter