Ch. 17 - Board Problems

① WHAT ARE THE ABSOLUTE MAX AND MIN VALUES on THE INTERVAL [0,3]

FOR. $f(x) = \dfrac{1}{2-x}$

② WHERE IS THE LOCAL MINIMUM OF $f(x) = 3x^4 - 4x^3 + 2$?

③ FIND ALL INFLECTION POINTS FOR: $f(x) = \dfrac{x^4}{12} - \dfrac{x^3}{3}$

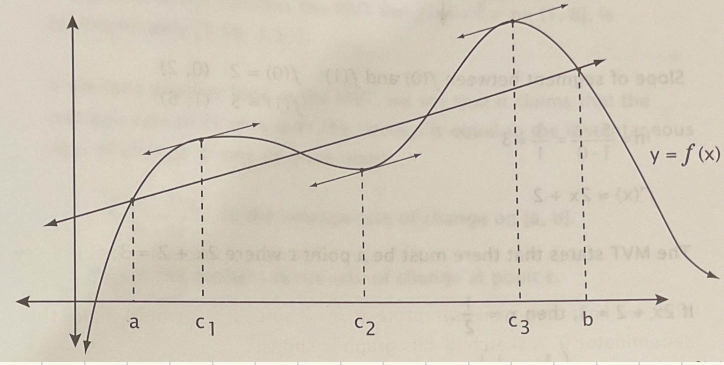
④ Find y' for $y = \sin(\tan^2(3x^2 - 6))$

Ch. 17 - MEAN VALUE THEOREM: L'HÔPITAL'S RULE

MEAN VALUE THEOREM (MVT)

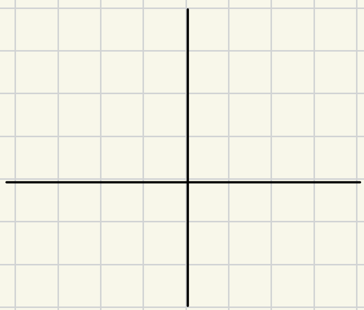
WHEN A CONTINUOUS GRAPH OF $y = f(x)$ OVER $[a, b]$ EXISTS AND IS DIFFERENTIABLE on (a, b), there MUST EXIST AT LEAST ONE NUMBER c between a and b such that the tangent line at c is parallel to chord $\overline{ab}$.

Figure 1



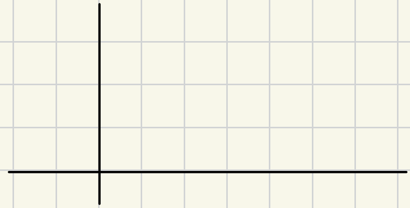
$\boxed{\text{EX. 1}}$ FIND ANY VALUE OF C WHICH SATISFIES THE MVT for $f(x) = x^2 + 2x + 2$ on $[0, 1]$

$f(x) = x^2 + 2x + 2$



EX. 2

FIND ANY VALUE OF c WHICH SATISFIES THE MVT FOR $f(x) = \sqrt[3]{x}$ on $[1, 8]$



EX. 3

Buddy Quick arrived at the toll booth after traveling two hours on a road where the speed limit was 60 mph. He had covered 150 miles. He was cited for speeding. Use the MVT to explain why.

Rolle's Theorem:
In Rolle's Theorem, f(a) = f(b) = 0. The conclusion is that there exists at least one point c in (a,b) where f'(c) = 0

Rolle's theorem is used in advanced algebra classes to prove the existence of a root of a polynomial.

EX. 4

SHOW THAT $x - \frac{3}{x} = 0$ HAS EACTLY 1 REAL SOLUTION ON $[1, 3]$.

L'Hôpital's (or L'Hôspital's) Rule connects limits with derivatives.

L'Hôpital's Rule (LR)
Suppose f(a) = g(a) = 0 and f'(a) and g'(a) exist with g'(a) not equal to 0, then:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(a)}{g'(a)}$$

$\boxed{\text{Ex. 5}}$ $$\lim_{x \to 0} \frac{3x + \sin(x)}{2x}$$

$f(x) = 3x + \sin x$ so $f(0) =$
$g(x) = 2x$ so $g(0) =$

DIFFERENTIATE EACH PART SEPARATELY,

$f'(x) =$
$g'(x) =$

$$\lim_{x \to 0} \frac{\quad}{\quad} =$$

L'HOPITAL'S RULE CAN BE EXTENDED TO LIMITS THAT INVOLVE $\frac{\infty}{\infty}$

★ THIS RULE CAN BE APPLIED AS MANY TIMES AS NECESSARY.

$\boxed{\text{Ex. 6}}$ $\lim_{x \to \infty} \frac{\ln^2(x+1)}{2x^3} = \_\_\_\_$

APPLY LR. $f'(x) =$ $\quad\quad \therefore \lim_{x \to \infty} \_\_\_\_\_\_\_\_$

$g'(x) =$

NOTE: YOU CAN ONLY APPLY LR WHEN LIMIT IS $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Odd and Even Functions
A function f(x) is an even function if f(-x) = f(x) for all x in the domain.
A function f(x) is an odd function if f(-x) = -f(x) for all x in the domain.

$\boxed{\text{EX. 7}}$ IS $f(x) = \frac{2}{x^2}$ = even, odd, or neither

$\boxed{\text{EX. 8}}$ is $f(x) = x^3 + \frac{1}{x}$ even, odd, neither?

$\boxed{\text{EX. 9}}$ is $f(x) = 2 - \frac{1}{x}$ even, odd, neither?

Find the value(s) of c, if any, that satisfies the MVT for the function and interval shown.

1. $f(x) = \sqrt{x}$ [1, 4]

2. $f(x) = x^3 + 3x$ [−1, 1]

3. $f(x) = \frac{1}{2x}$ [1, 3]

4. $f(x) = x^2 - 4$ [−3, −1]

5. It took five hours for the temperature to rise from 2°F to 22°F. Explain why there was a point in time where the temperature was rising at exactly 4°F per hour.

6. Speeding tickets are written for anyone who is traveling over 10 mph above the posted speed limit. A driver covered 200 miles on the turnpike in 2 3/4 hours. The speed limit was 55 mph. Should the driver receive a ticket? Explain.

LESSON PRACTICE 17A

Find the limits.

7. $\lim_{x \to 3} \frac{x-3}{x^2-9}$

8. $\lim_{x \to 0} \frac{\sin(4x)}{x}$

9. $\lim_{\theta \to 0} \frac{2\cos(\theta)-2}{\theta^2}$

10. $\lim_{\alpha \to 0} \frac{2(-\sin(\alpha)+\alpha)}{\alpha^2}$

11. $\lim_{x \to \infty} \frac{2x}{e^x}$

12. $\lim_{x \to 0} \frac{\sin^2(x)}{1-\cos(x)}$