Lesson 7 Imaginary and Complex Numbers

So far in algebra we have encountered positive numbers and negative numbers. Both of these are real. There is another option that arises when working with squares and square roots. (+3) x (+3) = +9 or $(+3)^2$ = +9. (-3) x (-3) = +9 or $(-3)^2$ = +9. The converse of squaring a number is finding the square root of a number. $\sqrt{9}$ is either +3 or -3, and we can write this as ±3, read "plus or minus 3". So, whether we are squaring a positive number or a negative number, the answer is a positive number. $(-3)^2$ and $(+3)^2$ both equal 9. So what happens when we encounter $\sqrt{-9}$? I like to separate this using what we know about multiplying radicals: $\sqrt{-9} = \sqrt{9}\sqrt{-1}$. Now we have $\sqrt{9}$, which we can solve. The new concept revolves around $\sqrt{-1}$. Since there is no real number which when squared equals (-1), we call this an imaginary number or imaginary unit, and refer to it as " i " (small letter i). (In electrical engineering, capital I represents current, so j is used in that application instead to represent imaginary numbers, to keep from confusing the two). What is interesting is that even though i is imaginary, i^2 is not. Remember that $\sqrt{4}\sqrt{4} = \sqrt{16} = 4$, so $\sqrt{4}\sqrt{4} = 4$ or $\sqrt{-7}\sqrt{-7} = -7$, or $\sqrt{X}\sqrt{X} = X$, so $\sqrt{-1}\sqrt{-1} = -1$. Another way to write this with the symbol i representing $\sqrt{-1}$ is $i^2 = -1$.

A complex number is a combination of a real number and an imaginary number. It is similar to a mixed number, which is a number and a fraction. Some examples of complex numbers are 4 + 3i and 27 - 9i. Imaginary numbers can be used in all the basic operations. Treat i as a radical or a variable.

Example 1

$$3i + 5i = 8i \text{ or } \sqrt{-9} + \sqrt{-25} = \sqrt{9}\sqrt{-1} + \sqrt{25}\sqrt{-1} = 3i + 5i = 8i$$

Example 2

$$7i - 4i = 3i$$

In the next example, remember that you can only combine or compare two numbers that are the same kind or value.

Example 3

$$7i + 5i = 12i, \quad 7i + 5 = 7i + 5$$

You cannot combine 7i and 5 because they are not the same kind. 7i is an imaginary number while 5 is a real number.

Example 4

$$(4i)(3i) = 12i^2 = 12(-1) = -12 \quad \text{Remember } i^2 = -1$$

Another way to write this is:

$$(4i)(3i) = (\sqrt{16}\sqrt{-1})(\sqrt{9}\sqrt{-1}) = \sqrt{144}(-1) = 12(-1) = -12$$

Example 5

$$\sqrt{-121} = \sqrt{121}\sqrt{-1} = 11i$$

Example 6

$$\sqrt{-64} + \sqrt{-49} = 8i + 7i = 15i$$

Example 7

$$i \cdot i \cdot i \cdot i = i^2 \cdot i^2 = (-1)(-1) = 1$$

Example 8

$$(2\sqrt{-3})(8\sqrt{-3}) = 16(-3) = -48$$

Practice Problems

Simplify each in terms of i:

1) $\sqrt{-36}$ 2) $\sqrt{-169}$ 3) $\sqrt{-225}$ 4) $\sqrt{-81X^2Y^2}$ 5) $\sqrt{-9/49}$ 6) $\sqrt{-27X^5}$

Simplify each, and then combine like terms.

7) $\sqrt{-16}+\sqrt{-25}=$

8) $\sqrt{-4}+\sqrt{-144}=$

9) $\sqrt{-169}-2\sqrt{-25}=$

10) $3\sqrt{-27}-4\sqrt{-8}=$

11) $\sqrt{-100}-3\sqrt{-16}=$

12) $4\sqrt{196}+3\sqrt{289}=$

13) $i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i =$

14) $\sqrt{-7}\cdot\sqrt{-14}=$

15) $(-8i)(5i)=$

16) $i^3 =$

17) $(9i)\sqrt{-64}=$

18) $\sqrt{-11}\sqrt{-11}=$

19) $(i^2)^3 =$

20) $(7\sqrt{-5})(4\sqrt{-5})=$

Solutions

1) $\sqrt{-36}=\sqrt{36}\sqrt{-1}=6i$

2) $\sqrt{-169}=\sqrt{169}\sqrt{-1}=13i$

3) $\sqrt{-225}=\sqrt{225}\sqrt{-1}=15i$

4) $\sqrt{-81X^2Y^2}=\sqrt{81}\sqrt{-1}\sqrt{X^2}\sqrt{Y^2}=9XYi$

5) $\sqrt{-9/49}=\sqrt{9/49}\sqrt{-1}=\frac{3}{7}i$

6) $\sqrt{-27X^5}=\sqrt{27}\sqrt{X^4}\sqrt{X}\sqrt{-1}=3X^2i\sqrt{3X}$ (You will often see this written as $3X^2\sqrt{3X}\,i$. Either order has the same value.)

7) $\sqrt{-16}+\sqrt{-25}=4i+5i=9i$

8) $\sqrt{-4}+\sqrt{-144}=2i+12i=14i$

9) $\sqrt{-169}-2\sqrt{-25}=13i-2(5i)=3i$

10) $3\sqrt{-27}-4\sqrt{-8}=3(3\sqrt{3})i-4(2\sqrt{2})i=9i\sqrt{3}-8i\sqrt{2}$

11) $\sqrt{-100}-3\sqrt{-16}=10i-3(4i)=10i-12i=-2i$

12) $4\sqrt{196}+3\sqrt{289}=4\text{x}14+3\text{x}17=107$

13) $i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i = i^2 \cdot i^2 \cdot i^2 \cdot i^2 = (-1)(-1)(-1)(-1) = 1$

14) $\sqrt{-7}\cdot\sqrt{-14}=i\sqrt{7}\cdot i\sqrt{14}=i^2\sqrt{98}=-7\sqrt{2}$

15) $(-8i)(5i)=-40(i^2)=-40(-1)=40$

16) $i^3=i^2\cdot i=(-1)\cdot i=-i$

17) $(9i)\sqrt{-64}=9i\cdot 8i=72i^2=-72$

18) $\sqrt{-11}\sqrt{-11}=-11$

19) $(i^2)^3=(-1)^3=-1$

20) $(7\sqrt{-5})(4\sqrt{-5})=28(-5)=-140$