Lesson 6   Fractional Exponents and Solving Equations with Rational Expressions

As we learned in Algebra 1, square roots and cube roots can also be written as exponents. Instead of $\sqrt{5}$ we can write $5^{1/2}$.   The cube root of 7 can be written as $\sqrt[3]{7}$ or $7^{1/3}$.

Two or more operations can be expressed as one fractional exponent. The square root of 9 raised to the third power is written as: $(\sqrt{9})^3 = (9^{1/2})^3 = 9^{3/2} = 27$.   The square root of 9 is 3, and 3 to the third power is 27.

Example 1 $(\sqrt{4})^5 = (4^{1/2})^5 = 4^{5/2} = 32$

$\searrow = 2^5 = 32$

The square root of 4 is 2, and 2 to the fifth power is 32.

Example 2 $(\sqrt[3]{27})^2 = (27^{1/3})^2 = 27^{2/3} = 9$

$\searrow = (3)^2 = 9$

The cube root of 27 is 3, and 3 to the second power is 9.

The denominator of the fractional exponent tells what root, and the numerator tells to what power it is raised.

Example 3 $(\sqrt{\sqrt{5}}) = (5^{1/2})^{1/2} = 5^{1/4}$

Example 4 $(\sqrt{\sqrt{81}}) = (81^{1/2})^{1/2} = 81^{1/4} = 3$

$\searrow = \sqrt{9} = 3$

The square root of 81 is 9, and the square root of 9 is 3. So the square root of the square root of 81 is 3.

Practice Problems

1) $(8^{1/3})^2 =$

2) $(3^4)^{1/2} =$

3) $(32^{1/5})^3 =$

4) $(81^{1/4})^3 =$

Rewrite using fraction exponents, then solve.

5) $(\sqrt[4]{16})^3 =$

6) $(\sqrt{3^4}) =$

7) $(\sqrt{4})^3 =$

8) $(\sqrt{25})^3 =$

9) $(\sqrt{\sqrt[3]{X}}) =$

10) $(\sqrt{\sqrt{Y}}) =$

11) $(\sqrt[3]{Q^4}) =$

12) $(\sqrt[3]{A^2}) =$

Solutions

1) $(8^{1/3})^2 = 2^2 = 4$

2) $(3^4)^{1/2} = 3^{4/2} = 3^2 = 9$
$\searrow = 81^{1/2} = 9$

3) $(32^{1/5})^3 = 32^{3/5} = 8$
$\searrow = (2)^3 = 8$

4) $(81^{1/4})^3 = (3)^3 = 27$

5) $(\sqrt[4]{16})^3 = (16^{1/4})^3 = 2^3 = 8$

6) $(\sqrt{3^4}) = (3^4)^{1/2} = 81^{1/2} = 9$

7) $(\sqrt{4})^3 = (4^{1/2})^3 = 2^3 = 8$

8) $(\sqrt{25})^3 = (25^{1/2})^3 = 5^3 = 125$

9) $(\sqrt{\sqrt[3]{X}}) = (X^{1/3})^{1/2} = X^{1/6}$

10) $(\sqrt{\sqrt{Y}}) = (Y^{1/2})^{1/2} = Y^{1/4}$

11) $(\sqrt[3]{Q^4}) = (Q^4)^{1/3} = Q^{4/3}$

12) $(\sqrt[3]{A^2}) = (A^2)^{1/3} = A^{2/3}$