Lesson 3 Scientific Notation and Combining Like Terms

Scientific notation is used in science to solve equations with very large and/or very small numbers. If we were asked to compute 200 times the distance from the earth to the sun (which is 93 million miles, or 93,000,000), using the normal method of multiplying, it would require a good deal of paper and pencil, and have lots of zeroes.

$$\begin{array}{r} 93{,}000{,}000 \\ \underline{\times\ 200} \end{array}$$

Scientific notation provides an easier and more efficient method. It is closely related to exponential notation.

93,000,000 in exponential notation is $9 \times 10^7 + 3 \times 10^6$. In scientific notation you keep the 9 and the 3 together, instead of separating them, and place the decimal point so the 9 is in the units place. Then choose the exponent figured from the number in the units place. The 3 is forgotten when choosing the exponent.

Example 1 Multiply 93,000,000 times 200.

93,000,000 in scientific notation is 9.3×10^7 200 in scientific notation is 2×10^2

To solve our original problem of 200 times the distance from the sun to the earth, multiply the numbers times the numbers & the exponents times the exponents.

$$(9.3 \times 10^7)(2 \times 10^2) = (9.3 \times 2)(10^7 \times 10^2) = (18.6)(10^9) = (1.86 \times 10^1)(10^9) = 1.86 \times 10^{10}$$

Example 2 Multiply 1,900 x 50.

1,900 = 1.9×10^3 in scientific notation 50 in scientific notation is 5×10^1

Multiply the numbers times the numbers & the exponential terms times the exponential terms.

$$(1.9 \times 10^3)(5 \times 10^1) = (1.9 \times 5)(10^3 \times 10^1) = (9.5)(10^4)$$

You can also show very small decimal numbers with scientific notation. Remember that you may only have one number in the units place and the rest as decimals. Study the examples:

Example 3 Change .000054 to Scientific Notation

$$.000054 = 5.4 \times 10^{-5} = 5.4 \times \frac{1}{10^5} \text{ or } 5.4 \times \frac{1}{100{,}000}$$

Example 4 Multiply 30,000,000 x .000023.

30,000,000 = 3.0×10^7 in scientific notation .000023 in scientific notation is 2.3×10^{-5}

$$(3 \times 10^7)(2.3 \times 10^{-5}) = (3 \times 2.3)(10^7 \times 10^{-5}) = (6.9)(10^2) = 6.9 \times 10^2$$

Example 5 Divide 500,000 ÷ 8,000

500,000 = 5.0×10^5 in scientific notation 8,000 in scientific notation is 8.0×10^3

$$(5 \times 10^5) \div (8 \times 10^3) = (5 \div 8)(10^5 \div 10^3) = (.625)(10^2) = (6.25 \times 10^{-1}) \times 10^2 = 6.25 \times 10^1$$

Practice Problems

1) 93,000,000 x .000054 =

2) 18,000 x .007 =

3) 640,000 x .92 =

4) 12,400 ÷ .04 =

5) 40,000 x 3,000 ÷ 60 =

6) .00058 x .0023 =

Solutions

1) $(9.3 \times 5.4)(10^7 \times 10^{-5}) = 50.22 \times 10^2 = (5.022 \times 10^1) \times 10^2 = 5.022 \times 10^3$

2) $(1.8 \times 7)(10^4 \times 10^{-3}) = 12.6 \times 10^1 = (1.26 \times 10^1) \times 10^1 = 1.26 \times 10^2$

3) $(6.4 \times 9.2)(10^5 \times 10^{-1}) = 58.88 \times 10^4 = (5.888 \times 10^1) \times 10^4 = 5.888 \times 10^5$

4) $(1.24 \div 4)(10^4 \div 10^{-2}) = .31 \times 10^6 = (3.1 \times 10^{-1}) \times 10^6 = 3.1 \times 10^5$

5) $(4 \times 3 \div 6)(10^4 \times 10^3 \div 10^1) = 2.0 \times 10^6$

6) $(5.8 \times 2.3)(10^{-4} \times 10^{-3}) = 13.34 \times 10^{-7} = (1.334 \times 10^1) \times 10^{-7} = 1.334 \times 10^{-6}$

Even though to be in the proper form, the digit must be in the units place, there are times when doing larger problems that it is advantageous to leave the numbers larger, and make the necessary corrections in the exponents. This allows for reducing or dividing by a common factor (sometimes called cancelling), and saves steps in problem solving. Example 6 employs this technique. You can do the long method to compare the results if you wish.

Example 6

$$\frac{(2{,}700)(3{,}500)}{(9{,}000{,}000)} = \frac{(27 \times 10^2)(35 \times 10^2)}{(90 \times 10^5)}$$

$$\frac{(\overset{3}{\cancel{27}} \times 10^2)(35 \times 10^2)}{(\underset{10}{\cancel{90}} \times 10^5)}$$ You can divide 27 and 90 by 9.

$$\frac{(\overset{3}{\cancel{27}} \times 10^2)(\overset{7}{\cancel{35}} \times 10^2)}{(\underset{2\,\cancel{10}}{\cancel{90}} \times 10^5)}$$ Then you can divide 35 and 10 by 5.

$$\frac{(3 \times 10^2)(7 \times 10^2)}{(2 \times 10^5)} = \frac{(3 \times 7)(10^2 \times 10^2)}{(2 \times 10^5)} = (21)(10^4) \div (2 \times 10^5)$$

$$= (21 \div 2)(10^{4-5})$$

$$= (10.5)(10^{-1}) = (1.05 \times 10^1)(10^{-1})$$

$$= 1.05$$

Combining Like Terms One of the key concepts we have focused on since the beginning is that numbers tell us how many, and place value tells what kind or what value. Building on this, we found that you can only combine or compare things that are the same kind, or value.

The terms and expressions in *Algebra 2* are becoming more complex, but the concepts remain the same. What we need to learn in this lesson is how to identify which terms are the same kind, then combine them. The strategies to be employed are not new either. When the terms are rational expressions, the first step is to reduce or simplify as much as possible. When exponents are present, either make them all positive, or put all on the same line by changing the signs of the exponent (opposite sign, opposite place). Some of the problems look tricky, but after patiently applying these strategies, we can distinguish between the apples and the oranges and combine the apples with the apples and the oranges with the oranges. Let's do a few examples.

Example 1

$$\frac{30X^{-1}}{10} + 2\,^2Y^2 - \frac{2}{X} + \frac{2Y}{Y^{-1}}$$

$$3X^{-1} + 4Y^2 - \frac{2}{X} + \frac{2Y}{Y^{-1}}$$ Step #1 - Simplify what you can.

$$\frac{3}{X} + 4Y^2 - \frac{2}{X} + 2Y^2 = \frac{1}{X} + 6Y^2$$ Step #2 - Make all exponents positive, or

or

$$3X^{-1} + 4Y^2 - 2X^{-1} + 2Y^2 = X^{-1} + 6Y^2$$ put all the variables on one line.

Example 2

$$3XY^{-2} - 4X^2Y^2 + \frac{7X^2}{XY^2}$$

$$3XY^{-2} - 4X^2Y^2 + \frac{7X}{Y^2}$$ Step #1 - Simplify what you can.

$$\frac{3X}{Y^2} - 4X^2Y^2 + \frac{7X}{Y^2} = \frac{10X}{Y^2} - 4X^2Y^2$$ Step #2 - Make all exponents positive, or

or

$$3XY^{-2} - 4X^2Y^2 + 7XY^{-2} = 10XY^{-2} - 4X^2Y^2$$ put all the variables on one line.

Practice Problems *Simplify and combine like terms.*

1) $\frac{5B^{-1}}{A^{-1}} - \frac{7B^2A^2}{B^3A^1} + \frac{3B^3B^0}{A^1A^{-2}} =$

2) $\frac{5X^4X^{-1}}{X^2Y^2} - 2X^2Y^{-2} + \frac{6X^4Y^2X^{-1}}{XY^4} =$

3) $8XXY - YXXY + \frac{2XY}{X^{-1}} =$

4) $4X^2X^{-1} - \frac{12X^2}{X^3} + 8X =$

5) $6A^2B^{-2}A - \frac{4AB^2B^{-1}}{A^{-1}B^3} + \frac{9B^3B^{-3}}{A^{-3}AB^2} =$

6) $9X^{-2}A^{-2}X + X^3AA^{-1} + \frac{7A^3X^0}{A^2X^3}$

Solutions

1) $5AB^{-1} - 7B^{-1}A + 3B^3A$

$= -2B^{-1}A + 3B^3A$

2) $\frac{5X}{Y^2} - \frac{2X^2}{Y^2} + \frac{6X^2}{Y^2}$

$= \frac{5X}{Y^2} + \frac{4X^2}{Y^2}$

3) $8X^2Y - X^2Y^2 + 2X^2Y$

$= 10X^2Y - X^2Y^2$

4) $4X - 12X^{-1} + 8X$

$= 12X + 12X^{-1}$

5) $6A^3B^{-2}\ 4A^2B^{-2} + 9A^2B^{-2}$

$= 6A^3B^{-2} + 5A^2B^{-2}$

6) $9X^{-1}A^{-2} + X^3 + \frac{7A}{X^3}$