Lesson 13   Discriminants

In the quadratic formula, what goes on underneath the radical sign indicates what kind of roots, or what the nature of the solutions will be. The solutions fall into three categories; real and rational (3, 1/4, .75), imaginary (4 + 7i, 3i), or real and irrational ($\sqrt{2}$).

The key part of the quadratic formula which helps us discriminate what kind of roots we have is B2 - 4AC, which is under the radical sign. If this a negative number, for example, then the solution will obviously be imaginary because you are asked to find the square root of a negative number. In the examples below, keep an eye on the B2 - 4AC and how it affects the final solution. By the way, this key part of the quadratic formula, for its ability to discriminate the type of root, is called the <u>discriminant</u>.

Example 1 $X^2 + 7X + 10 = 0$

$A = 1, B = 7, \& C = 10$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-7 \pm \sqrt{(7)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$X = \frac{-7 \pm \sqrt{49 - 40}}{2} = \frac{-7 \pm \sqrt{9}}{2} = \frac{-7 + 3}{2} \text{ or } \frac{-7 - 3}{2}$$

$$X = \frac{-4}{2}, \frac{-10}{2} = -2, -5$$

B2 - 4AC = 9 in this example, and the roots (-2 & -5) are <u>real</u> and <u>rational</u>. A rational number is a number that can be expressed as a ratio or fraction. Notice that 9 is a perfect square. If the discriminant is a perfect square, then your roots will be real, not imaginary.

Example 2 $X^2 + 3X - 3 = 0$

$A = 1, B = 3, \& C = -3$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(3) \pm \sqrt{(3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$X \frac{-3 \pm \sqrt{9 + 12}}{2} = \frac{-3 \pm \sqrt{21}}{2} = \frac{-3 + \sqrt{21}}{2}, \frac{-3 - \sqrt{21}}{2}$$

B 2 - 4AC = 21 in this example, and the roots are real and irrational. An irrational number, the square root of 21 in this example, cannot be expressed as a ratio or a fraction, but it is real.

Example 3 $X^2 - 10X + 25 = 0$

$A = 1, B = -10, \& C = 25$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 25}}{2 \cdot 1} = \frac{10 \pm \sqrt{100 - 100}}{2} = \frac{10}{2} = 5$$

B 2 - 4AC = 0 in this example. The roots are real and rational, and they are equal. If this polynomial, which is a perfect square, had been factored, there would have been two identical solutions (5). This is called a double root. When the discriminant is 0, you can expect a double root.

$$X^2 - 10X + 25 = 0$$

$$(X-5)(X-5) = 0$$

$$X-5=0 \qquad X-5=0$$
$$X=5 \qquad\quad X=5$$

Example 4 $X^2 - 5X + 8 = 0$

A = 1, B = -5, & C = 8

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1}$$

$$X = \frac{5 \pm \sqrt{25 - 32}}{2}$$

$$X = \frac{5 \pm \sqrt{-7}}{2} = \frac{5 + i\sqrt{7}}{2} \text{ or } \frac{5 - i\sqrt{7}}{2}$$

$B^2 - 4AC = -7$ in this example, and the roots are imaginary since they include an "i".

In summary:

1) If the discriminant is a perfect square, the roots are: Real, Rational, & Unequal.
2) If the discriminant is positive, >0, and not a perfect square, the roots are: Real, Irrational, & Unequal.
3) If the discriminant is equal to 0, the roots are: Real, Rational, & Equal; a Double Root.
4) If the discriminant is negative, <0, the roots are: Imaginary.

Practice Problems   *Predict the nature of the solutions, then solve to find the exact roots. Factor when possible.*

1) $5X^2 + 2X = 2X + 45$

2) $X^2 + 16 = -8X$

3) $X^2 = 2X - 5$

4) $X^2 - 2/3X = 4/3$

5) $2X^2 = X + 3$

Solutions

1) $5X^2 + 2X = 2X + 45$

$5X^2 + 2X - 2X - 45 = 0$

$5X^2 - 45 = 0$

$X^2 - 9 = 0$

A=1, B=0, C=-9

$B^2 - 4AC = 0^2 - (4)(1)(-9) = 36$

Disc. = 36, a perfect square
Roots as in #1 in summary.

$X^2 - 9 = 0$
$(X-3)(X+3) = 0$

X-3=0   X+3=0
X=3   X=-3

(3, -3)

2) $X^2 + 16 = -8X$

$X^2 + 8X + 16 = 0$

A=1, B=8, C=16

$B^2 - 4AC = (8)^2 - (4)(1)(16) = 0$

Disc. = 0
Double roots as in #3.

$X^2 + 8X + 16 = 0$
$(X+4)(X+4) = 0$

X+4=0   X+4=0
X=-4   X=-4

(-4, -4)

3) $X^2 = 2X - 5$

$X^2 - 2X + 5 = 0$

A=1, B=-2, C=5

$B^2 - 4AC = (-2)^2 - (4)(1)(5) = -16$

Disc. < 0
Imaginary roots as in #4.

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

(1+2i, 1-2i)

4) Disc. > 0, as in #2 in summary.

$$\left(\frac{1}{3} + \frac{\sqrt{13}}{3}, \frac{1}{3} - \frac{\sqrt{13}}{3} \right)$$

5) Disc. = 25, a perfect square
Roots as in #1 in summary.

(3/2, -1)