

Ch. 29 - Age and Boat Problems

Board Problem

1. There are 38 coins, quarters and dimes that total \$6.80. How many of each?
2. There are three even integers such that five times the first is four less than three times the third integer. Find the integers.
3. Brand X sells 21oz bags of mixed nuts that contain 29% peanuts. To make Brand X, Brand A (35% peanuts) and Brand B (25% peanuts) are combined. How many of each brand do they need to make X?

Ch.29 - AGE & BOAT PROBLEMS

Ex. 1 - In 9 years, Steve will be twice as old as Isaac. 4 years ago, Isaac was $\frac{1}{3}$ the age of Steve.

How old are they now?

Lesson 29 Age and Boat in the Current Problems

As in previous problems, we'll be using substitution and elimination to solve the equations that we come up with. With most word problems, the key is setting them up properly, then choosing the appropriate variables.

Age Problems The most effective way to learn problems of this sort is to study an example.

Example 1 In 9 years Steve will be twice as old as Isaac. Four years ago, Isaac was $\frac{1}{3}$ the age of Steve. How old are they now?

Two pieces of information were given, so we make two equations from them. S will represent the age of Steve now, and I will represent Isaac's age. The first statement speaks of 9 years from now, so that is $S + 9$ and $I + 9$, and the equation is $(S+9) = (I + 9) \times 2$. Read the information again as you look at the equation. The second statement is 4 years ago, so $S-4$ and $I - 4$ represent four years ago. The equation is: $\frac{1}{3} (S - 4) = (I - 4)$.

Equation 1 $S + 9 = 2 (I + 9)$

$$S + 9 = 2 I + 18$$

$$S = 2 I + 9$$

Equation 2 $I - 4 = \frac{1}{3} (S - 4)$

$$3(I - 4) = S - 4$$

$$3I - 12 = S - 4$$

$$3I - 12 = (2 I + 9) - 4$$

$$3I - 12 = 2 I + 5$$

$$I = 17$$

$$S + 9 = 2 (17 + 9)$$

$$S = 43$$

Substitute S from Equation 1 into Equation 2.

If $I = 17$, then put this in either equation to find S .

Practice Problems

- 1) In 7 years Joseph will be twice as old as Emmitt. Last year Emmitt was $\frac{1}{6}$ the age of Joseph. How old are they now?
- 2) In 10 years Sandi Beth will be twice as old as Ethan. Five years ago Sandi Beth was $3 \frac{1}{2}$ the age of Ethan. How old are they now?

Ch. 29 - BOAT PROBLEMS

MORE $D = RT$

$$R_{\text{DOWNSTREAM}} = R_{\text{BOAT}} + R_{\text{CURRENT}}$$

$$R_{\text{UPSTREAM}} = R_{\text{BOAT}} - R_{\text{CURRENT}}$$

$$\rightarrow \textcircled{1} \quad \underline{D_D = R_D t_D} \quad \text{so ...} \quad D_D = (R_B + R_C) t_d$$

$$\rightarrow \textcircled{2} \quad D_U = R_U t_U \quad \text{so ...} \quad D_U = (R_B - R_C) t_u$$

Jeff's boat travels 65 mi downstream, in 4 hrs. His boat travels upstream 20 mi in 5 hours. What is the speed of the boat (R_b) and the speed of the current (R_c)?

Gateway Clipper can go 12 mi upstream in the same amount of time it takes to go 24 miles downstream. The Allegheny River flows at 3mph. What is the rate of the boat?

Boat in the Current If you have ever canoed, you know that it is a pleasurable experience with the current, but a chore to paddle against the current. For the sake of clarity, whenever we say downstream, we mean with the current, while upstream denotes against the current. If you happen to be in a stream with a 3 mph current, and you just lie back and look at the sky, you will move along at 3 mph as well. If you decide to paddle at 5 mph, then you will be going 8 mph, 3 mph from the current and 5 mph from paddling.

$$\text{Rate}_{\text{downstream}} = \text{Rate}_{\text{boat}} + \text{Rate}_{\text{water}}$$

$$R_D = R_B + R_W$$

$$8 \text{ mph} = 5 \text{ mph} + 3 \text{ mph}$$

If you are traveling upstream at a rate of 5 mph in the same current, this is what the equation would look like.

$$\text{Rate}_{\text{upstream}} = \text{Rate}_{\text{boat}} - \text{Rate}_{\text{water}}$$

$$R_U = R_B - R_W$$

$$2 \text{ mph} = 5 \text{ mph} - 3 \text{ mph}$$

Another way to write these two equations is: $R_D = B + W$ and $R_U = B - W$.

Remember from the motion problems that $D = RT$. In the example, if the rate downstream is 8 mph, $B + W = 5 + 3$, and if we were paddling for 3 hours, then distance downstream equals rate downstream multiplied by time downstream, or:

$$D_D = R_D \times T_D$$

$$D_D = 8 \text{ m/h} \times 3 \text{ h}$$

$$24 \text{ m} = 8 \text{ m/h} \times 3 \text{ h}$$

With this equation from motion and the one we just learned, we can put them together and form two other equations.

$$R_D = B + W$$

$$R_U = B - W$$

$$D_D = R_D \times T_D$$

$$D_U = R_U \times T_U$$

$$D_D = (B + W) T_D$$

$$D_U = (B - W) T_U$$

Example 2 Jeff's boat travels 64 miles downstream in 4 hours. The same boat travels 20 miles upstream in 5 hours. What is the speed of the boat and the current?

$$D_D = R_D \times T_D$$

$$64 = R_D \times 4$$

$$16 = R_D$$

$$R_D = B + W$$

$$16 = B + W$$

$$D_U = R_U \times T_U$$

$$20 = R_U \times 5$$

$$4 = R_U$$

$$R_U = B - W$$

$$4 = B - W$$

$$16 = B + W$$

$$4 = B - W$$

$$20 = 2B$$

$$10 = B$$

$$4 = 10 - W$$

$$6 = W$$

The rate of the boat is 10 mph and the rate of the current is 6 mph.

Example 3 The Gateway Clipper can go 12 miles upstream in the same time as it takes to go 24 miles downstream. The Allegheny river flows at 3 mph. What is the rate of the boat? In this problem time is the same, so there is no need for subscripts on the T.

$$\begin{array}{rcl}
 D_D = R_D T_D & & D_U = R_U T_U \\
 D_D = (B + W) T & & D_U = (B - W) T \\
 24 = (B + 3) T & & 12 = (B - 3) T \\
 24 = BT + 3T & & 12 = BT - 3T \\
 -12 = -BT + 3T & \xleftarrow{\times(-1)} & \\
 \hline
 12 = 6T & & \\
 2 = T & \longrightarrow & 24 = (B + 3) T \\
 & & 24 = (B + 3)2 \\
 & & 12 = B + 3 \\
 & & 9 = B
 \end{array}$$

The rate of the boat is 9 mph, the rate of the current is 3 mph, and the time is 2 hours.

Practice Problems

- 1) The Delta Queen traveled at a steady rate of 15 mph without the current. It took her 8 hours to travel downstream to the port and 16 hours to travel upstream to her home port. How far did she travel to the port, and what is the speed of the water?
- 2) The Deerslayer and the Mohican chief were paddling furiously to escape the Iroquois. They paddled the canoe at a steady rate for 60 miles in water with a rate of 5 mph. When it was safe, they paddled back upstream at the same rate and went 6 miles in 3 hours. What is the rate of the canoe? How long did they travel downstream?

LESSON PRACTICE

29A

Answer the questions.

1. Richard is five times as old as David. In five years, Richard will be three times as old as David. How old are Richard and David now?
2. Next year, Joanne will be six times Nicole's age. In seven years, Nicole will be one-third the age of Joanne. How old is each girl now?
3. Two years ago, Tyler's dad was two times Tyler's age. In 23 years, Tyler will be two-thirds the age of his dad. How old is each one now?
4. In five years, Carol will be six times the age of Heather. In two more years after that, Heather will be one-fifth the age of Carol. How old are Carol and Heather now?

5. A boat travels at a rate of 12 mph without any current. The boat went downstream for two hours, and then turned around and returned to the starting point in six hours. How far did the boat travel each way, and what is the speed of the current?
6. A bird flew with the wind for three hours and covered a distance of 36 miles. Then it turned around and flew against the wind, going two miles in one hour before stopping to rest. What was the speed of the bird and the speed of the wind?
7. A barge can go 21 miles downstream in the same time it goes 9 miles upstream. The rate of the water is 2 mph. What is the rate of the barge?
8. An airplane flew between two cities at a speed of 200 mph. With the wind behind the plane, the trip took 9 hours. The return trip was against the wind and took 11 hours. What was the distance between the cities and the speed of the wind?

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